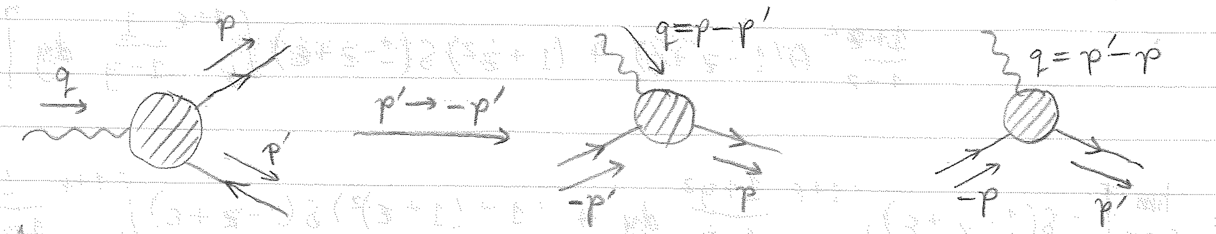
Evaluation of $\hat{H}^{\gamma q}_{\Sigma,V}$

$$p\cdot p' = \frac{Q^2}{2z}$$

Use $e^+e^- \longrightarrow q\bar{q}$ result:

Relabel $p \leftrightarrow p'$



Average over initial spin/colors

$$H^{\mu\nu} = \frac{-1}{2N_c} Q_i^2 N_c \left(1 + 2\,\mathrm{Re}[\delta F_1(-Q^2)]\right) 4\left(-p^\mu p'^\nu - p^\nu p'^\mu + g^{\mu\nu} p\cdot p'\right)$$

$$= 2Q_i^2\left(1 + 2\,\mathrm{Re}[\delta F_1(-Q^2)]\right)\left(p^\mu p'^\nu + p^\nu p'^\mu - g^{\mu\nu} p\cdot p'\right)$$

$$-g_{\mu\nu} H^{\mu\nu} = 2Q_i^2\left(1 + 2\,\mathrm{Re}[\delta F_1(-Q^2)]\right)\left(-2p\cdot p' + d\,p\cdot p'\right)$$

Recall $p\cdot p' = \dfrac{-q^2}{2z} = \dfrac{Q^2}{2z}$

$$= \cancel{2}Q_i^2 (d-2)\frac{Q^2}{\cancel{2}z}\left(1 + 2\,\mathrm{Re}[\delta F_1(-Q^2)]\right)$$

$$= 2Q_i^2(1-\epsilon)\frac{Q^2}{z}\left(1 + 2\,\mathrm{Re}[\delta F_1(-Q^2)]\right)$$

Then, using $d(\mathrm{LIPS})_1 = 2\pi \dfrac{z}{Q^2}\delta(1-z)$

$$4\pi\,\hat{H}^{\gamma q}_{\Sigma,V} = -g_{\mu\nu}\int H^{\mu\nu}\, d(\mathrm{LIPS})_1$$

$$= 2Q_i^2(1-\epsilon)\frac{\cancel{Q^2}}{\cancel{z}}\left(1 + 2\,\mathrm{Re}[\delta F_1(-Q^2)]\right) 2\pi \frac{\cancel{z}}{\cancel{Q^2}}\delta(1-z)$$

$$\Rightarrow \hat{H}^{\gamma q}_{\Sigma,V} = Q_i^2(1-\epsilon)\,\delta(1-z)\left(1 + 2\,\mathrm{Re}[\delta F_1(-Q^2)]\right)$$

$$= Q_i^2(1-\epsilon)\,\delta(1-z)\left\{1 + \cancel{2}\frac{\alpha_s C_F}{\underset{2}{\cancel{4}}\pi}\left(\frac{4\pi\mu^2}{Q^2}\right)^\epsilon \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left(\frac{-2}{\epsilon^2} - \frac{3}{\epsilon} - \left(8 + \frac{\pi^2}{3}\right)\right)\right\}$$

$$= Q_i^2(1-\epsilon)\,\delta(1-z)\left\{1 - \frac{\alpha_s}{2\pi} C_F\left(\frac{4\pi\mu^2}{Q^2}\right)^\epsilon \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + 8 + \frac{\pi^2}{3}\right)\right\}$$

Adding the two contributions (real and virtual corrections) together,

$$\hat{H}_{\Sigma}^{\gamma q} \equiv \hat{H}_{\Sigma,R}^{\gamma q} + \hat{H}_{\Sigma,V}^{\gamma q} = Q_i^2(1-\epsilon)\left\{\delta(1-z) + \frac{\alpha_s}{2\pi}C_F\left(\frac{4\pi\mu^2}{Q^2}\right)^{\epsilon}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\right.$$

$$\times\left[\left(\frac{-3}{2\epsilon} - \frac{9}{2} - \frac{\pi^2}{3}\right)\delta(1-z) - \frac{1}{\epsilon}\frac{1+z^2}{(1-z)_+} + (1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ - \frac{1+z^2}{1-z}\ln(z)\right.$$

$$\left.\left. - \frac{3}{2}\frac{1}{(1-z)_+} + (3-z) + O(\epsilon)\right]\right\}$$

Organize by powers of ϵ

$$\hat{H}_{\Sigma}^{\gamma q} = Q_i^2(1-\epsilon)\left\{\delta(1-z) + \frac{\alpha_s}{2\pi}C_F\left(\frac{4\pi\mu^2}{Q^2}\right)^{\epsilon}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left[\overbrace{-\left(\frac{3}{2}\delta(1-z) + \frac{1+z^2}{(1-z)_+}\right)}^{\left(\frac{1+z^2}{1-z}\right)_+ \text{ see facing page}}\frac{1}{\epsilon}\right.\right.$$

$$\left.\left. - \left(\frac{9}{2} + \frac{\pi^2}{3}\right)\delta(1-z) + (1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ - \frac{1+z^2}{1-z}\ln(z) - \frac{3}{2}\frac{1}{(1-z)_+} + (3-z)\right]\right\}$$

$$\hat{H}_{\Sigma}^{\gamma q} = Q_i^2(1-\epsilon)\left\{\delta(1-z) + \frac{\alpha_s}{2\pi}\left[-C_F\left(\frac{1+z^2}{1-z}\right)_+\left(\frac{1}{\epsilon} - \gamma_E + \ln 4\pi - \ln\left(\frac{Q^2}{\mu^2}\right)\right)\right.\right.$$

$$\left.\left. C_F\left(-\left(\frac{9}{2} + \frac{\pi^2}{3}\right)\delta(1-z) + (1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ - \frac{1+z^2}{1-z}\ln(z) - \frac{3}{2}\frac{1}{(1-z)_+} + (3-z)\right)\right]\right\}$$