Relationship of invariant variables to LAB frame variables

Easy: just insert definition of 4-vectors in LAB frame.

$$s = (P+k)^2 = M^2 + m^2 + 2P\cdot k$$
$$= M^2 + m^2 + 2ME$$

inverse relation:

$$E = \frac{s - M^2 - m^2}{2M}$$

(very simple; independent of other kinematic variables)

$$Q^2 = -2m^2 + 2(EE' - |\vec{k}||\vec{k}'|\cos\theta)$$

if $m=0$, it simplifies:

$$= 4EE'\sin^2(\theta/2) = 2EE'(1-\cos\theta)$$

$$W^2 = (P-q)^2 = M^2 - Q^2 - 2P\cdot q$$
$$= M^2 - 4EE'\sin^2(\theta/2) - 2M(E-E')$$

$$x = \frac{Q^2}{2P\cdot q} = \frac{Q^2}{2M(E-E')} = \frac{2EE'\sin^2(\theta/2)}{M(E-E')}$$

$$y = \frac{q\cdot P}{k\cdot P} = \frac{M(E-E')}{ME} = \frac{E-E'}{E}$$

$$\nu = P\cdot q = M(E-E')$$