Interrelationships of invariant variables

$$[Q^2, W^2] \longleftrightarrow [Q^2, x]$$

$$W^2 = (P+q)^2 = P^2 + q^2 + 2P\cdot q = M^2 - Q^2 + \frac{Q^2}{x} \quad \text{with } x = \frac{Q^2}{2P\cdot q}$$

$$\longrightarrow: \quad \boxed{W^2 = Q^2\left(\frac{1-x}{x}\right) + M^2} \qquad [1]$$

inverse relation:

$$\longleftarrow: \quad \boxed{x = \frac{Q^2}{(W^2 - M^2) + Q^2}} \qquad [2]$$

Notice: elastic limit $W^2 = M^2$ corresponds to $x \to 1$

$$[Q^2, x] \longleftrightarrow [x, y]$$

$$y = \frac{q\cdot P}{k\cdot P} = \frac{Q^2}{2x\ k\cdot P} \quad \text{with } x = \frac{Q^2}{2P\cdot q}, \quad s = (k+P)^2 = m^2 + M^2 + 2k\cdot P$$

$$\longleftarrow: \quad \boxed{y = \frac{Q^2}{x(s - M^2 - m^2)}} \qquad [3]$$

inverse relation:

$$\longrightarrow: \quad \boxed{Q^2 = xy\,(s - M^2 - m^2)} \qquad [4]$$

$$[Q^2, W^2] \longleftrightarrow [x, y]$$

[4] $\quad \boxed{Q^2 = xy\,(s - M^2 - m^2)}$ [4]

[4]→[1] $\quad \boxed{W^2 = (1-x)y\,(s - M^2 - m^2) + M^2}$ plug [4] into [1]

inverse relations:

[2]→ $\quad \boxed{x = \frac{Q^2}{(W^2 - M^2) + Q^2} \quad \text{and} \quad y = \frac{W^2 - M^2 + Q^2}{s - M^2 - m^2}} \qquad [5]$