Factorization of N-particle Veneziano amplitude – for spinless pole

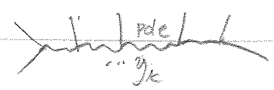
$$B_N = \int_0^1 dy_2 \cdots dy_{N-2} \prod_{i=2}^{N-2} y_i^{-1-\alpha(s_{1i})} \prod_{m=2}^{N-2} \prod_{n=m+1}^{N-1} x_{mn}(y)^{-1-\alpha(y_{mn})} \prod_{j=2}^{N-2} (1-y_j y_{j+1})^{-1}$$

Suppose we take S_{1k} such that $\alpha(S_{1k}) \to 0$.

Then: y_k integration diverges in the lower limit.

$\Rightarrow$ expand in plus dist: $\dfrac{1}{y^{1+\alpha(s_{ik})}} = \dfrac{-1}{\alpha(s_{ik})}\delta(y_k) + \ldots$

pole in the k^{th} internal line.



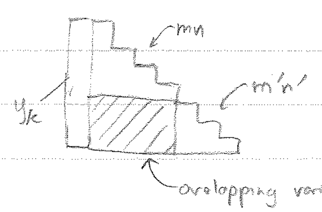
Then, write

$$\prod_{i=2}^{N-2} y_i^{-1-\alpha(s_{1i})} \to \frac{-1}{\alpha(s_{ik})}\delta(y_k) \prod_{i=2}^{k-1} y_i^{-1-\alpha(s_{1i})} \prod_{i'=k+1}^{N-2} y_{i'}^{-1-\alpha(s_{1i'})} + \ldots$$

All (mn) channel variables overlapping with 1k: $x_{mn} \to 1$.

$$\prod_{m=2}^{N-2}\prod_{n=m+1}^{N-1} x_{mn}^{-1-\alpha(s_{mn})} \longrightarrow \prod_{m=2}^{k-1}\prod_{n=m+1}^{k} x_{mn}^{-1-\alpha(s_{mn})} \prod_{m'=k+1}^{N-2}\prod_{n'=m'+1}^{N-1} x_{m'n'}^{-1-\alpha(s_{m'n'})}$$

See: channel variables list:



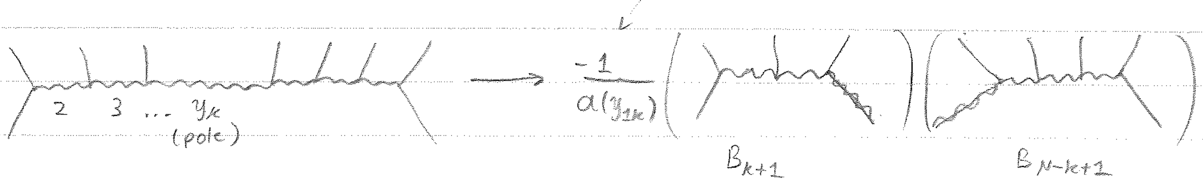
and finally,

$$\prod_{j=2}^{N-2}(1-y_j y_{j+1})^{-1} \longrightarrow \prod_{j=2}^{k-2}(1-y_j y_{j+1})^{-1} \prod_{j'=k+1}^{N-3}(1-y_{j'} y_{j'+1})^{-1}$$

So that B_N factorizes at the pole (integrate over y_k)

$$\boxed{B_N = \frac{-1}{\alpha(S_{1k})} B_{k+1}(S_{<k})\, B_{N-k+1}(S_{>k}) + \text{reg.}}$$

residue ~ $P_{l=0}(t_k)$ (spin-0 exch.)



(Bootstrap condition)

– hard to show full factorization for higher poles $\longrightarrow$ operator formalism.

Possible to show residue at higher poles have the correct polynomial order:

Suppose s_{1k} is such that $\alpha(s_{1k}) \to \ell > 0$ (integer).

Isolate y_k integration variable

$$B_N = \int_0^1 dy_k \; y_k^{-1-\alpha(s_{1k})} \underbrace{\left[\text{stuff} \equiv F(y_k) \right]}_{\substack{\text{series expand this around } y_k \approx 0. \\ \text{(same as isolating poles in Beta function)}}}$$

$$= \int_0^1 dy_k \frac{1}{y_k^{1+\alpha(s_{1k})}} \left[\cdots + \frac{1}{\ell!} \frac{\partial^\ell}{\partial y_k^\ell} F(y_k) \Big|_{y_k=0} y_k^\ell + \cdots \right]$$

$$= \int_0^1 dy \underbrace{\frac{1}{y_k^{1+\alpha(s_{1k})-\ell}}}_{\frac{-1}{\alpha(s)-\ell}\delta(y)} \frac{1}{\ell!} \frac{\partial^\ell}{\partial y_k^\ell} F(y_k) \Big|_{y_k=0}$$

$$\frac{\partial}{\partial y} x_{mn}(y)^{-1+\alpha(s_{mn})}$$

Taking ℓ derivatives will bring down at most ℓ powers of $-1-\alpha(s_{jk})$

$\nwarrow$ dual variables.

$$= \frac{-1}{\alpha(s)-\ell} \times \left(\text{polynomial in } s_{jk} \text{ of degree at most } \ell \right).$$

$\Rightarrow$ no ancestors.