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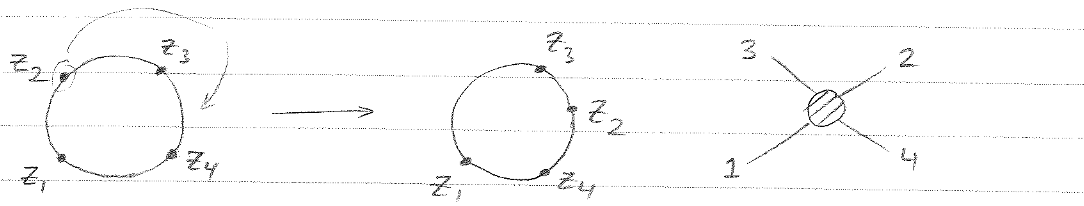
Extended integration range



This diagram is expected to display poles in u & t.

Start with

$$\int_1^\infty dz \; |z|^{-1-\alpha(s)} \; |1-z|^{-1-\alpha(t)}$$

ch. integration variables:

$$y = 1 - \frac{1}{z} \qquad \begin{array}{l} z: [1, \infty] \\ y: [0, 1] \end{array}$$

$$\Rightarrow \quad z = \frac{1}{1-y} \qquad dz = \frac{dy}{(1-y)^2}$$

$$= \int_0^1 \frac{dy}{(1-y)^2} \left| \frac{1}{1-y} \right|^{-1-\alpha(s)} \left| \frac{-y}{1-y} \right|^{-1-\alpha(t)}$$

Write: $\quad -\alpha(s) = \alpha' s - \alpha_0 \qquad\qquad s+t+u = \sum m^2$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad s = -t-u-4\alpha_0/\alpha'$

$$= \alpha' t + \alpha' u + 4\alpha_0 - \alpha_0$$

$$= \alpha(t) + \alpha(u) + \alpha_0$$

$$= \int_0^1 \frac{dy}{(1-y)^2} \left| \frac{1}{1-y} \right|^{-1+\alpha(t)+\alpha(u)+\alpha_0} \left| \frac{-y}{1-y} \right|^{-1-\alpha(t)}$$

$$= \int_0^1 \cancel{\frac{dy}{(1-y)^2}} \cancel{\left| \frac{1}{1-y} \right|^{-2}} \left| \frac{1}{1-y} \right|^{\alpha(u)+\alpha_0} \underset{\text{remove}}{|-y|}^{-1-\alpha(t)}$$

$$= \int_0^1 dy \; |y|^{-1-\alpha(t)} \; |1-y|^{-\alpha_0-\alpha(u)} \quad = B_4(t,u). \quad ✓$$

$$\boxed{\text{set } \alpha_0 = 1}$$