Factorization of N-particle amplitude

- poles for a given Reggeon appear entirely in the $\hat{D}(s)$ operator:

$$\hat{D}(s) = \int_0^1 dy \underbrace{(1-y)^{-1+\alpha_0}}_{\text{Use binomial theorem}} y^{-1-\alpha(s)} y^{\sum_n(-n a_n^\dagger a_n)}$$

Use binomial theorem
extended to continuous powers — I. Newton

$$(1-x)^{\alpha_0-1} = \sum_{r_0=0}^{\infty} \binom{\alpha_0-1}{r_0}(-y)^{r_0}$$

$$= \sum \binom{\alpha_0-1}{r_0}(-1)^{r_0} y^{r_0}$$

$$= \sum \binom{r_0-\alpha_0+\cancel{1}-\cancel{1}}{r_0} y^{r_0}$$

identity:

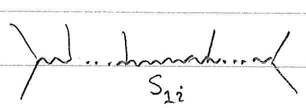
$$(-1)^k\binom{n}{k} = \binom{k-n-1}{k}$$

$$\hat{D}(s) = \sum_{r_0=0}^{\infty}\binom{r_0-\alpha_0}{r_0}\int_0^1 dy\, y^{-1-\alpha(s)+\sum(-n a_n^\dagger a_n)+r_0}$$

$$= \sum_{r_0=0}^{\infty}\binom{r_0-\alpha_0}{r_0}\frac{1}{-\alpha(s)+\sum_n(-n a_n^\dagger a_n)+r_0}$$

Note: if $\alpha_0=1$, only $r_0=0$ term survives $\binom{-1}{0}=1$

Plug into amplitude



$$= \langle 0|\hat{V}(p_{N-1})\ldots \hat{D}(s_{1i})\ldots \hat{V}(p_2)|0\rangle$$

$$= \langle 0|\hat{V}(p_{N-1})\ldots\sum_{r_0=0}^{\infty}\binom{r_0-\alpha_0}{r_0}\frac{1}{-\alpha(s_{1i})+\sum(-n a_n^\dagger a_n)+r_0}\ldots\hat{V}(p_2)|0\rangle$$

Insert pair of covariantized completeness relation around propagator

$$\sum_{\{r\}}(-1)^{r_n^\mu}\,|\{r\}\rangle\langle\{r\}| = 1$$

contravariant excitations — covariant excitations

$$= \sum_{\{r\}} \sum_{\{r'\}} \langle 0 | \hat{V}(p_{N-1}) \cdots | \{r\}\rangle\langle\{r\}| \sum_{r_0=0}^{\infty} \binom{r_0-\alpha_0}{r_0} \frac{1}{-\alpha(s_{1i}) + \sum_n (-n \underset{\text{act}}{a_n^\dagger \cdot a_n}) + r_0} |\{r'\}\rangle\langle\{r'\}| \cdots \hat{V}(p_2)|0\rangle$$

Lorentz indices implied

Now, $\sum_n (-n a_n^\dagger a_n)|\{r'\}\rangle = \underbrace{\sum_n n \sum_\mu r_n'^{\mu}}_{\text{abbr: } = \bar{r}} |\{r'\}\rangle$

Then $\langle\{r\}|\{r'\}\rangle = (-1)^{r_n^\mu} \delta_{\{r_n^\mu\}\{r_n'^\mu\}}$.

— sum over r'.

$$\boxed{B_N = \underbrace{\sum_{\{r\}} \sum_{r_0=0}^{\infty}}_{\substack{\text{Sum such}\\ \text{that } \alpha(s_{1i}) = \bar{r}+r_0}} (-1)^{r_n^\mu} \binom{r_0-\alpha_0}{r_0} \underbrace{\langle 0 | \hat{V}(p_{N-1}) \cdots |\{r\}\rangle}_{\text{Momenta from right half of diagram}} \underbrace{\frac{1}{-\alpha(s_{1i}) + \bar{r} + r_0}}_{\text{Regge pole}} \underbrace{\langle\{r\}| \cdots \hat{V}(p_2)|0\rangle}_{\text{Momenta from left half of diagram}}}$$

FACTORIZED!

<u>Regge poles</u>

$\alpha(s_{1i}) = R =$ positive integer $\Rightarrow$ pole for $\bar{r} + r_0 = R$

There will be a separate contribution to pole for each configuration $\{r\}$ and r_0 such that they add up to R.

Each contribution is labeled by occupation numbers $(r_1, r_2, \ldots)$ and r_0 "quantum numbers"

<u>Note:</u>

Unitarity allows us to interpret $B_{\{r_0,r\}\to X} = \sqrt{\binom{r_0-\alpha_0}{r_0}} \langle 0 | \hat{V}(p_{N-1}) \ldots |\{r\}\rangle$

as amplitude for particle with quantum numbers $\{r_0, r\}$ to decay into scalar particles.

Similarly, $B_{X' \to \{r_0,r\}} = \sqrt{\binom{r_0-\alpha_0}{r_0}} \langle\{r\}| \ldots \hat{V}(p_2)|0\rangle$ is the amplitude to produce particle with quantum numbers $\{r_0, r\}$.

$\rightarrow$ <u>Next</u>: Investigate residue, and show $\{r_0, r\}$ is not a state of definite spin.