Properties of Virasoro operators

① $\hat{L}_n f(\hat{L}_0) = f(\hat{L}_0 + n)\hat{L}_n$

Proof: Write as Taylor series

$$\hat{L}_n f(\hat{L}_0) = \hat{L}_n \sum_{m=0}^{\infty} \hat{L}_0^m \frac{1}{m!} f^{(m)}(0)$$

Push $\hat{L}_n$ through $\hat{L}_0 \cdots \hat{L}_0$:

$$\hat{L}_n \hat{L}_0 = (\hat{L}_0 + n)\hat{L}_n$$

$$\Rightarrow \hat{L}_n \underbrace{\hat{L}_0 \cdots \hat{L}_0}_{m} = (\hat{L}_0 + n)^m \hat{L}_n$$

so,

$$\hat{L}_n f(\hat{L}_0) = \sum_{m=0}^{\infty} (\hat{L}_0 + n)^m \frac{1}{m!} f^{(m)}(0)\, \hat{L}_n$$

$$= f(\hat{L}_0 + n)\hat{L}_n \quad \checkmark$$

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