Residues of Veneziano B_4 function

$$B_4(s,t) = \sum_{n=0}^{\infty} \frac{R_n(-\alpha(t))}{n-\alpha(s)}, \qquad R_n(-\alpha(t)) = \frac{1}{n!}(1+\alpha(t))(2+\alpha(t))\ldots(n+\alpha(t))$$

$$\alpha(t) = \alpha' t + \alpha_0 \quad \leftarrow \text{Write in terms of } z = \cos\theta$$

$$= \frac{-\alpha_0}{m^2} t + \alpha_0 \quad \text{(Bootstrap condition)}$$

Equal mass kinematics: $t = -\frac{1}{2}(1-z)(s-4m^2)$

But $\alpha(s) = \alpha' s + \alpha_0 = n$

$$\frac{-\alpha_0}{m^2} s + \alpha_0 = n \quad \Rightarrow \quad s = m^2\left(1 - \frac{n}{\alpha_0}\right)$$

So
$$t = -\frac{1}{2}(1-z)\left(m^2\left(1-\frac{n}{\alpha_0}\right) - 4m^2\right)$$
$$= \frac{1}{2}(1-z)\, m^2 \left(3 + \frac{n}{\alpha_0}\right)$$

$$\therefore \quad \alpha(t) = \frac{-\alpha_0}{2\cancel{m^2}}(1-z)\cancel{m^2}\left(3+\frac{n}{\alpha_0}\right) + \alpha_0$$
$$= -\frac{1}{2}(1-z)(3\alpha_0 + n) + \alpha_0$$

Then
$$R_n(z) \equiv R_n(-\alpha(t)) = \frac{1}{n!}\prod_{j=1}^{n}\left[j + \alpha(t)\right]$$
$$= \frac{1}{n!}\prod_{j=1}^{n}\left[j - \frac{1}{2}(1-z)(3\alpha_0+n) + \alpha_0\right]$$

Residue function in terms of Regge intercept

In terms of $P_\ell(z)$ [Mathematica]

$$R_{n=0}(-\alpha(t)) = P_0(z)$$

$$R_{n=1}(-\alpha(t)) = \frac{1}{2}(1-\alpha_0)P_0(z) + \frac{1}{2}(3\alpha_0+1)P_1(z)$$

$$R_{n=2}(-\alpha(t)) = \frac{1}{12}(6\alpha_0^2 + 3\alpha_0 + 2)P_0(z) - \frac{1}{4}(\alpha_0 - 1)(3\alpha_0+2)P_1(z) + \frac{1}{12}(3\alpha_0+2)^2 P_2(z)$$

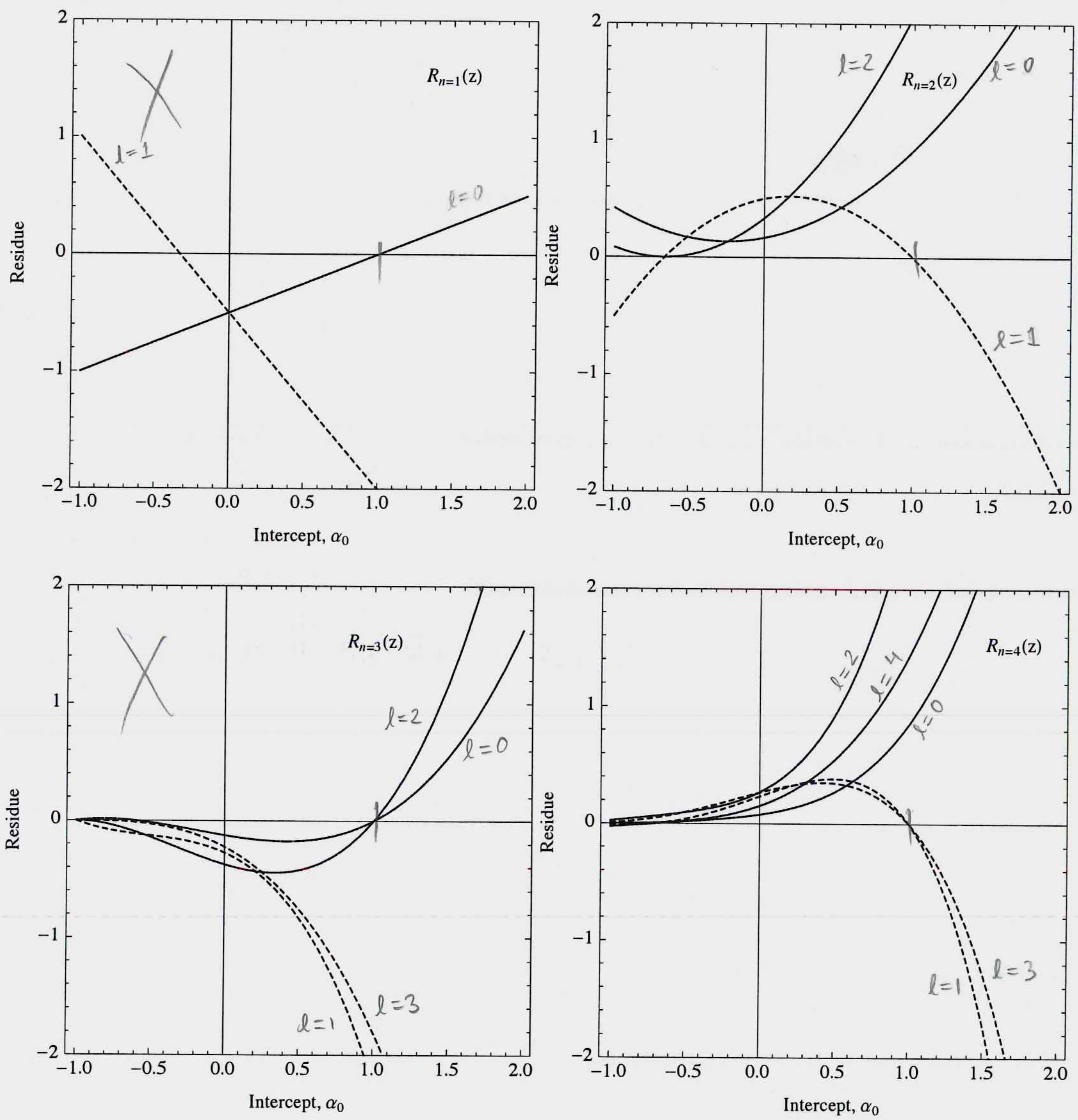
$$R_{n=3}(-\alpha(t)) = \frac{1}{24}(1-\alpha_0^2)(5\alpha_0+3)P_0(z) + \frac{1}{40}(\alpha_0+1)(21\alpha_0^2 + 12\alpha_0 + 11)P_1(z)$$
$$+ \frac{3}{8}(1-\alpha_0)(\alpha_0+1)^2 P_2(z) + \frac{9}{40}(\alpha_0+1)^3 P_3(z)$$

To satisfy unitarity, coeff. of all $P_\ell(z)$ must be greater than or equal to zero.

Residues of partial wave poles

– negative residue = ghost

– Wrong for $n=1$ & 3



Notice for $\alpha_0 = 1$: even n (right) $\Rightarrow$ odd ℓ residues vanish
even ℓ residues positive ✓

odd n (left) $\Rightarrow$ even ℓ residues vanish
odd ℓ residues negative :(

(causes tachyon on leading trajectory)

But, when added $V = \underbrace{B_4(s,t) + B_4(s,u)}_{\text{residues for all odd } \ell \text{ cancel (vanish)}} + B_4(t,u)$

$\Rightarrow$ amplitude free of Ghosts.