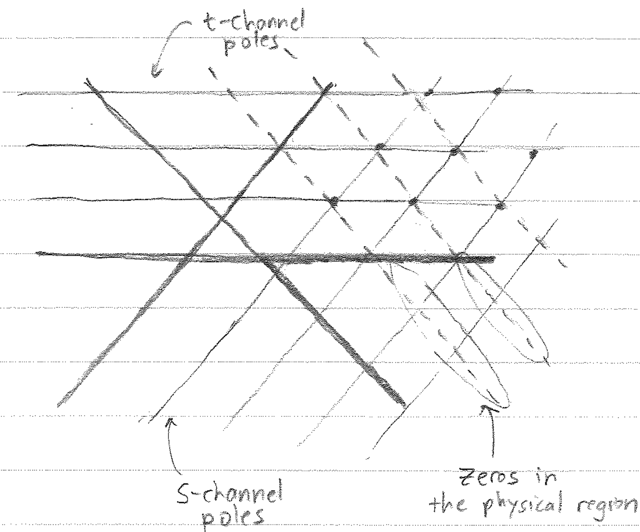
Zeros of the Veneziano amplitude



$$B_4(s,t) = \frac{\Gamma(-\alpha(s))\,\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))}$$

Denominator: $\Gamma(-\text{integer}) \rightarrow \frac{1}{0}$

$\Rightarrow$ Lines of $\alpha(s)+\alpha(t) = $ positive integer are zeros of $A(s,t)$.

In the s-t plane, (use $\alpha(s) = \alpha_0 + \alpha' s$):

$$2\alpha_0 + \alpha' \underbrace{(s+t)}_{-u-\sum m^2} = \text{positive integer}$$

$$\Rightarrow \quad u = \frac{2\alpha_0}{\alpha'} + \sum m^2 - \frac{(\text{int})}{\alpha'} = \text{constant}$$

Zeros are on lines of constant u, spaced by $(\alpha')^{-1} \approx 1\ \text{GeV}^2$

Do these zeros play any phenomenological role?

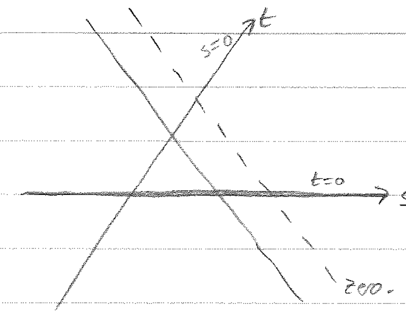
The full amplitude is $V(s,t) = g\left(A(s,t) + \underbrace{A(s,u) + A(t,u)}_{\text{these might remove zeros from } A(s,t).}\right)$

But, if u-channel is exotic such as in $K^- p \rightarrow \bar{K}^0 n$,
perhaps the full amplitude is just $V(s,t) = g\,A(s,t)$?

(Odorico 1971) - found fixed zeros along constant u at

$$u = -0.1,\ -0.7,\ -1.7\ \text{GeV}^2.$$

It is possible to fix the Regge trajectory by matching to <u>Adler's consistency condition</u>



"Adler's zero": $\pi\pi \to \pi\pi$ amplitude vanishes at off-shell point: $s = t = u = m_\pi^2$

match to first zero of Veneziano amplitude:

$$\alpha(s) + \alpha(t) = \text{positive integer} = 1$$

$$2\alpha_0 + \alpha'(s+t) = 1$$

Match to Adler's zero: $s = t = u = m_\pi^2$:

$$2\alpha_0 + 2\alpha' m_\pi^2 = 1 \quad \Rightarrow \quad \alpha_0 = \frac{1}{2} - \alpha' m_\pi^2 \qquad (*)$$

Now $\alpha(t) = \alpha_0 + \alpha' t$

$$= \frac{1}{2} - \alpha' m_\pi^2 + \alpha' t,$$

trajectory must reach $\alpha(t = m_\rho^2) = 1$ at rho meson mass (squared)

$$1 = \frac{1}{2} + \alpha'(m_\rho^2 - m_\pi^2)$$

$$\Rightarrow \alpha' = \frac{1}{2(m_\rho^2 - m_\pi^2)} = \frac{1}{2(0.775^2 - 0.135^2)}\,\text{GeV}^{-2} = \boxed{0.86\ \text{GeV}^{-2}\ \text{(slope)}}$$

$$(*) \Rightarrow \alpha_0 = \frac{1}{2} - (0.86\ \text{GeV}^{-2})(0.135\ \text{GeV})^2 = \boxed{0.48\ \text{(intercept)}}$$

Surprisingly good agreement with known ρ trajectory:

$$\alpha_\rho(t) = 0.5 + (0.9\ \text{GeV}^{-2})\,t$$