Symmetric form of the full Veneziano amplitude ($\alpha_0 = 1$ + bootstrap)

$$V(s,t,u) = \frac{-\pi g}{\Gamma(1+\alpha(t))\Gamma(1+\alpha(s))}\Bigg[\frac{\Gamma(1+\alpha(s)+\alpha(t))}{\sin \pi\alpha(t)}\,\frac{\sin(\pi\alpha(s)+\pi\alpha(t))}{\sin\pi\alpha(s)}$$

$$+\,\Gamma(\alpha(s)+\alpha(t)-\textstyle\sum\alpha)\left(\frac{\Gamma(1+\alpha(s))}{\sin\pi\alpha(t)\,\Gamma(\alpha(s)-\sum\alpha)}+\frac{\Gamma(1+\alpha(t))}{\sin\pi\alpha(s)\,\Gamma(\alpha(t)-\sum\alpha)}\right)\Bigg]$$

where $\sum\alpha = \alpha(s)+\alpha(t)+\alpha(u) = \underbrace{4\alpha' m^2}_{-\frac{\alpha_0}{m^2}\text{ Bootstrap condition}} + 3\alpha_0 = -\alpha_0$

Symmetric form obtained when $\alpha_0 = +1$. (save space: $\alpha(s)\equiv\alpha_s$, $\alpha(t)\equiv\alpha_t$, ...)

$$V(s,t,u) = \frac{-\pi g\,\Gamma(1+\alpha_s+\alpha_t)}{\Gamma(1+\alpha_s)\Gamma(1+\alpha_t)}\left[\frac{\sin(\pi\alpha_s+\pi\alpha_t)}{\sin(\pi\alpha_t)\sin(\pi\alpha_s)}+\frac{\cancel{\Gamma(1+\alpha_s)}}{\sin\pi\alpha_t\,\cancel{\Gamma(\alpha_s+1)}}+\frac{\cancel{\Gamma(1+\alpha_t)}}{\sin\pi\alpha_s\,\cancel{\Gamma(\alpha_t+1)}}\right]$$

use $\sin(a+b) = \sin a\cos b + \cos a \sin b$

$$= \quad '' \quad \left[\frac{\cos\pi\alpha_t}{\sin\pi\alpha_t}+\frac{\cos\pi\alpha_s}{\sin\pi\alpha_s}+\frac{1}{\sin\pi\alpha_t}+\frac{1}{\sin\pi\alpha_s}\right]$$

$$= \frac{-\pi g\,\Gamma(1+\alpha_s+\alpha_t)}{\Gamma(1+\alpha_s)\Gamma(1+\alpha_t)}\left[\frac{1+\cos\pi\alpha_s}{\sin\pi\alpha_s}+\frac{1+\cos\pi\alpha_t}{\sin\pi\alpha_t}\right]$$

$$\Gamma(1+\alpha) = \frac{1}{\Gamma(-\alpha)}\,\frac{\pi}{\sin(-\pi\alpha)}, \qquad \Gamma(1+\alpha_s+\alpha_t) = \Gamma(-\alpha_u)$$

$$V(s,t,u) = -\pi g\,\Gamma(-\alpha_s)\Gamma(-\alpha_t)\Gamma(-\alpha_u)\times\frac{\sin(-\pi\alpha_s)}{\pi}\,\frac{\sin(-\pi\alpha_t)}{\pi}\left[\frac{1+\cos\pi\alpha_s}{\sin\pi\alpha_s}+\frac{1+\cos\pi\alpha_t}{\sin\pi\alpha_t}\right]$$

$$= -\frac{g}{\pi}\Gamma(-\alpha_s)\ldots\Gamma(-\alpha_u)\left[\sin\pi\alpha_t(1+\cos\pi\alpha_s)+\sin\pi\alpha_s(1+\cos\pi\alpha_t)\right]$$

$$= \frac{-g}{\pi}\Gamma(-\alpha_s)\ldots\Gamma(-\alpha_u)\Big[\sin\pi\alpha_t+\sin\pi\alpha_s+\underbrace{\sin\pi\alpha_t\cos\pi\alpha_s+\sin\pi\alpha_s\cos\pi\alpha_t}_{\sin\pi(\alpha_t+\alpha_s)}\Big]$$

$$\sin\pi(\alpha_t+\alpha_s) = -\sin(\pi(\alpha_t+\alpha_s)+\pi) = +\sin(\pi(\underbrace{-\alpha_t-\alpha_s-1}_{\alpha_u}))$$

$$\boxed{V(s,t,u) = \frac{-g}{\pi}\Gamma(-\alpha_s)\Gamma(-\alpha_t)\Gamma(-\alpha_u)\left[\sin\pi\alpha_s+\sin\pi\alpha_t+\sin\pi\alpha_u\right]}$$