Shapiro-Virasoro form

Identity:

$$-4\cos\left(\frac{\pi\alpha_s}{2}\right)\cos\left(\frac{\pi\alpha_t}{2}\right)\cos\left(\frac{\pi\alpha_u}{2}\right) = \frac{-1}{2}e^{-\frac{i\pi}{2}(\alpha_s+\alpha_t+\alpha_u)}\left(1+e^{i\pi\alpha_s}\right)\left(1+e^{i\pi\alpha_t}\right)\left(1+e^{i\pi\alpha_u}\right)$$

[Use Mathematica: TrigToExp, Expand, Simplify]

Set $\alpha_s+\alpha_t+\alpha_u = -1 \quad (\alpha_0 = 1)$

$$= -\frac{1}{2}\underbrace{e^{i\pi/2}}_{i}\Big[1+e^{i\pi\alpha_s}+e^{i\pi\alpha_t}+e^{i\pi\alpha_u}+e^{i\pi\overbrace{(\alpha_s+\alpha_t)}^{-1-\alpha_u}} + e^{i\pi\underbrace{(\alpha_s+\alpha_u)}_{-1-\alpha_t}} + e^{i\pi\underbrace{(\alpha_t+\alpha_u)}_{-1-\alpha_s}} + \underbrace{e^{i\pi\overbrace{(\alpha_s+\alpha_t+\alpha_u)}^{-1}}}_{-1}\Big]$$

$$= -\frac{i}{2}\left[\cancel{1}+e^{i\pi\alpha_s}-e^{-i\pi\alpha_s}+e^{i\pi\alpha_t}-e^{-i\pi\alpha_t}+e^{i\pi\alpha_u}-e^{-i\pi\alpha_u}-\cancel{1}\right]$$

$$= \sin\pi\alpha_s + \sin\pi\alpha_t + \sin\pi\alpha_u .$$

Plug into $V(s,t,u)$:

$$V(s,t,u) = \frac{-g}{\pi}\Gamma(-\alpha_s)\Gamma(-\alpha_t)\Gamma(-\alpha_u)\left[-4\cos\left(\frac{\pi\alpha_s}{2}\right)\cos\left(\frac{\pi\alpha_t}{2}\right)\cos\left(\frac{\pi\alpha_u}{2}\right)\right]$$

Then, use:

$$\Gamma(-\alpha) = \frac{2^{-\alpha-1}}{\sqrt{\pi}}\Gamma\left(\frac{-\alpha}{2}\right)\Gamma\left(-\frac{\alpha}{2}+\frac{1}{2}\right)$$

AND

from $\Gamma(\alpha)\Gamma(1-\alpha) = \frac{\pi}{\sin\pi z}$

$$\Gamma\left(\frac{1}{2}+\frac{\alpha}{2}\right)\Gamma\left(\frac{1}{2}-\frac{\alpha}{2}\right) = \frac{\pi}{\cos\pi\alpha/2}$$

$$\Rightarrow \cos\left(\frac{\pi\alpha}{2}\right)\Gamma\left(\frac{1}{2}-\frac{\alpha}{2}\right) = \frac{\pi}{\Gamma\left(\frac{1}{2}+\frac{\alpha}{2}\right)}$$

so that

$$\cos\left(\frac{\pi\alpha}{2}\right)\Gamma(-\alpha) = \cos\frac{\pi\alpha}{2}\,\frac{2^{-\alpha-1}}{\sqrt{\pi}}\Gamma\left(\frac{-\alpha}{2}\right)\Gamma\left(\frac{-\alpha}{2}+\frac{1}{2}\right)$$

$$= \frac{2^{-\alpha-1}}{\sqrt{\pi}}\Gamma\left(\frac{-\alpha}{2}\right)\frac{\pi}{\Gamma\left(\frac{1+\alpha}{2}\right)}$$

$$= \frac{\sqrt{\pi}}{2^{\alpha+1}}\,\frac{\Gamma(-\alpha/2)}{\Gamma\left(\frac{1+\alpha}{2}\right)}$$

$$V(s,t,u) = \frac{4g}{\pi} \frac{\sqrt{\pi}}{2^{\alpha_s+1}} \frac{\Gamma(-\alpha_s/2)}{\Gamma\left(\frac{1+\alpha_s}{2}\right)} \frac{\sqrt{\pi}}{2^{\alpha_t+1}} \frac{\Gamma(-\alpha_t/2)}{\Gamma\left(\frac{1+\alpha_t}{2}\right)} \frac{\sqrt{\pi}}{2^{\alpha_u+1}} \frac{\Gamma(-\alpha_u/2)}{\Gamma\left(1+\frac{\alpha_u}{2}\right)}$$

$$= \frac{4\sqrt{\pi}\,g}{2^{\alpha_s+\alpha_t+\alpha_u+3}} \frac{\Gamma(-\alpha_s/2)\Gamma(-\alpha_t/2)\Gamma(-\alpha_u/2)}{\Gamma\left(\frac{1+\alpha_s}{2}\right)\Gamma\left(\frac{1+\alpha_t}{2}\right)\Gamma\left(\frac{1+\alpha_u}{2}\right)}$$

use $\alpha_s+\alpha_t+\alpha_u = -1$: Shapiro – Virasoro form

$$\boxed{V(s,t,u) = \sqrt{\pi}\,g \frac{\Gamma(-\alpha_s/2)\,\Gamma(-\alpha_t/2)\,\Gamma(-\alpha_u/2)}{\Gamma\left(-\frac{\alpha_t}{2}-\frac{\alpha_u}{2}\right)\Gamma\left(-\frac{\alpha_s}{2}-\frac{\alpha_u}{2}\right)\Gamma\left(-\frac{\alpha_s}{2}-\frac{\alpha_t}{2}\right)}}$$

Clear that numerator contains poles for even α_s, α_t, α_u only.

$\rightarrow$ odd daughter trajectories absent for $\alpha_0 = 1$.