Constructive definition of the effective potential

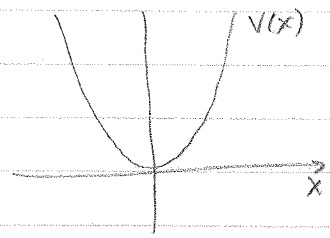
Consider a (0+1)-d field theory: $\underset{m^2 \to \omega^2}{\phi \to q \equiv x}$

$$L = \frac{1}{2}\dot{x}^2 - \frac{1}{2}\omega^2 x^2$$

$\downarrow$

$$H = \frac{1}{2}p^2 + \frac{1}{2}\omega^2 x^2$$

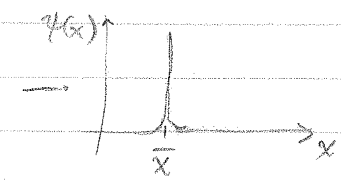
$$= -\frac{\hbar^2}{2}\frac{\partial^2}{\partial x^2} + \frac{1}{2}\omega^2 x^2$$



In classical mechanics, the potential may be naively defined as:
" given a particle at rest ($p=0$), at position $x = \bar{x}$, what is the energy of the system? $\Rightarrow$ Answer: $E = \cancelto{0}{T(p=0)} + V(\bar{x}) = V(\bar{x})$

Problem is that in QM, requiring a particle to be exactly at rest ($p=0$) and being precisely located at $x = \bar{x}$ is incompatible with the H. uncertainty relation. $\Delta x \Delta p \geq \frac{\hbar}{2}$

— Could use a wavefunction with definite position
(relaxing requirement that particle be at rest)
$\psi(x) \sim \sqrt{\delta(x-\bar{x})}$



But, then, the energy of the system will be infinite:

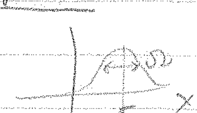
$$\langle \psi(x) | \hat{H} | \psi(x) \rangle = \underset{\infty}{\underset{\downarrow}{\langle T \rangle}} - \underset{V(\bar{x})}{\underset{\downarrow}{\langle U \rangle}}$$

Def^{n} in QM: given a particle that is approximately at rest, and approximately located at $x = \bar{x}$, what is the energy of the system?

$\longrightarrow$ How to define shape of such wavefunction?

There is the gaussian effective potential: fixed Ω

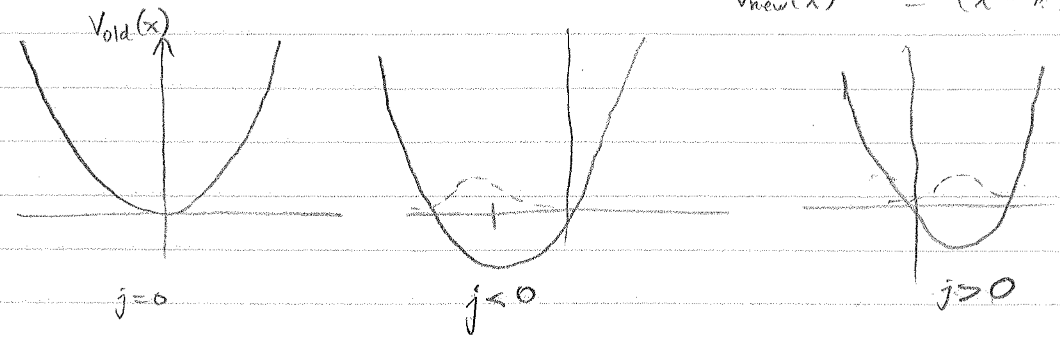
$$\psi = \mathcal{N} e^{-(x-\bar{x})^2/\Omega^2}$$



Coleman-Weinberg Eff. Potential:

construct state with the help of source $j(\bar{x})$.

Define a new Hamiltonian: $\hat{H} = \frac{1}{2}\hat{p}^2 + \underbrace{\frac{1}{2}\omega^2\hat{x}^2 - j\hat{x}}_{V_{new}(x)} = (x - \#)^2 + \#'$



Now, the shape of $\psi(x)$ to be used is the ground state of $\hat{H}_{new}$. The j to pick is the one that makes the ground state wavefunction localized at $\bar{x}$:

$$|\psi(x)\rangle_{j(\bar{x})},$$

Hence $V_{eff}(\bar{x}) = \langle\psi(x)|_{j(\bar{x})}\, \hat{H}\, |\psi(x)\rangle_{j(\bar{x})}$.

The source $j(\bar{x})$ is to be eliminated in favor of $\bar{x}$.