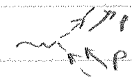
Diagrammatic evaluation of gauge contribution to V_{eff}:

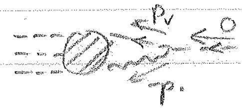
For convenience, put $\xi = 0$. Three reasons why $\xi = 0$ is popular.

[1] Lorenz gauges: $\partial_\mu A^\mu = 0$

$$D^{\mu\nu}(p) = \frac{-i}{p^2+i\epsilon}\left(+g^{\mu\nu} - (1-\xi)\frac{p^\mu p^\nu}{p^2+i\epsilon}\right)$$



$= ig(p+p')$

This way, diagrams like  vanish due to vertex!

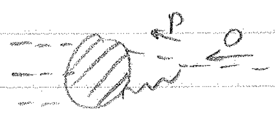
$$\sim \frac{-i\,(g^{\mu\nu} - p^\mu p^\nu/p^2)}{p^2+i\epsilon}\,(ip_\nu)\,\frac{i}{p^2-m^2+i\epsilon}$$

$= 0$; (ip_ν): vertex factor

In this gauge, scalars do not couple to ghosts.

– Same story for R_ξ gauges: $\langle\phi\rangle \to \phi_c$

$$D^{\mu\nu} = \frac{-i}{p^2-m^2+i\epsilon}\left(g^{\mu\nu} - (1-\xi)\frac{p^\mu p^\nu}{p^2-\xi m^2+i\epsilon}\right)$$

then 

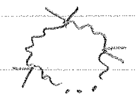
$$= \frac{-i}{p^2-m^2}\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)(ip_\nu)\frac{i}{p^2-m^2+i\epsilon}$$

[2] Even though there are scalar–ghost couplings, they are prop. to ξ, so setting $\xi = 0$ decouples them.

[3] The background R_ξ gauges and restricted R_ξ gauges coincide at $\xi = 0$.

$(\partial . A - \xi e \bar{\phi}\phi)$ $\qquad$ $(\partial . A - \xi e v \phi)$

$\boxed{\text{CAUTION}}$ The contributions to the effective potential that come from massless fields (at $\phi_c = 0$) — e.g. gauge fields — cannot be understood in terms of individual 1PI Feynman diagrams. The reason is the contribution from each one is IR divergent:

Typically,  $\sim \int \frac{d^d p}{(2\pi)^d} \left(\frac{1}{p^2}\right)^n \rightarrow$ IR divergent.

In the computation of the effective potential, the IR singularity is regulated by the background field, and upon summing over all 1PI functions, the result is well-behaved in the $\phi_c \to 0$ limit:

$$V_{1\text{-loop}}(\phi_c) = i \underbrace{\sum_{n=1}^{\infty}}_{\text{Perform sum}} \mu^{2\epsilon} \int \frac{d^d p}{(2\pi)^d} \frac{1}{2n} \left(\frac{g^2 \phi_c^2}{p^2 + i\epsilon}\right)^n \times \underbrace{\mathrm{Tr}\left[\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)\left(g^{\nu\rho} - \frac{p^\nu p^\rho}{p^2}\right) \cdots\right]}_{= d-1 \text{ (Landau gauge, } \xi = 0)}$$

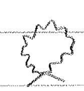
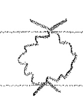
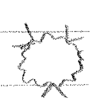
(this is analytic continuation)

$$= -(d-1)\frac{i}{2} \mu^{2\epsilon} \int \frac{d^d p}{(2\pi)^d} \ln\left(1 - \frac{g^2 \phi_c^2}{p^2 + i\epsilon}\right) \qquad \text{no longer IR divergent}$$

$$= \frac{-3}{64\pi^2} (g^2 \phi_c^2)^2 \left(\frac{1}{\epsilon} - \gamma_E + \ln 4\pi - \ln\left(\frac{g^2 \phi_c^2/2}{\mu^2}\right) + \frac{3}{2}\right)$$

well-behaved in $\phi_c \to 0$ limit.

Note: The diagrammatic viewpoint is equivalent to Taylor expanding $V_{\text{eff}}(\phi_c)$ about an IR logarithmic singularity at $\phi_c = 0$:

$V_{1\text{-loop}}(\phi_c) = -\Big[\frac{1}{2!}$  $\phi_c^2 + \frac{1}{4!}$  $\phi^4 + \frac{1}{6!}$  $\phi^6 + \ldots\Big]$

$$= -\left[\frac{1}{2!}(\infty_{UV} - \infty_{IR})\phi^2 + \frac{1}{4!}\infty_{IR}\phi^4 + \frac{1}{6!}\infty_{IR}\phi^6 + \ldots\right]$$

Take great care when studying $V_{\text{eff}}(\phi_c)$ in terms of individual diagrams.