One-loop Effective Potential in Scalar QED — Functional method

$$\mathcal{L} = \partial^\mu\phi^\dagger\partial_\mu\phi - m_R^2\phi^\dagger\phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$
$$- ie_R\mu^\epsilon A_\mu\phi^\dagger\overleftrightarrow{\partial^\mu}\phi + e_R^2\mu^{2\epsilon}A_\mu A^\mu\phi^\dagger\phi - \lambda_R\mu^{2\epsilon}(\phi^\dagger\phi)^2$$
$$+ \delta_\phi\partial^\mu\phi^\dagger\partial_\mu\phi - \delta_{m^2}\phi^\dagger\phi - \frac{1}{4}\delta_A F_{\mu\nu}F^{\mu\nu}$$
$$- i\delta_{A\phi\phi}\mu^\epsilon A_\mu\phi^\dagger\overleftrightarrow{\partial^\mu}\phi + \delta_{A^2\phi^2}\mu^{2\epsilon}A_\mu A^\mu\phi^\dagger\phi - \delta_{\phi^4}\mu^{2\epsilon}(\phi^\dagger\phi)^2.$$

Write ϕ in terms of real fields: $\phi \equiv \frac{1}{\sqrt{2}}(\Phi^1 + i\Phi^2)$ — ignore counterterms:

$$= \frac{1}{2}\partial_\mu\Phi^i\partial^\mu\Phi^i - \frac{1}{2}m_R^2\Phi^i\Phi^i + \frac{1}{2}A_\mu(\partial^2 g^{\mu\nu} + \partial^\mu\partial^\nu)A_\nu$$
$$+ e_R\mu^\epsilon A_\mu\epsilon^{ij}\Phi^i\partial^\mu\Phi^j + \frac{1}{2}e_R^2\mu^{2\epsilon}A_\mu A^\mu\Phi^i\Phi^i - \frac{\lambda_R}{4}\mu^\epsilon(\Phi^i\Phi^i)^2$$

Need $S_{New}[\phi_c;\phi,A] = S[\phi_c+\phi] - S[\phi_c] - \frac{\delta S[\phi_c]}{\delta\phi^i_{c,x}}\phi^i_x$

$$S[\phi_c+\phi] = \int d^dx\Big[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m_R^2(\phi_c+\phi)^2 + \frac{1}{2}A_\mu(\partial^2g^{\mu\nu}+\partial^\mu\partial^\nu)A_\nu$$
$$+ e_R\mu^\epsilon A_\mu\epsilon^{ij}(\phi_c+\phi)^i\partial^\mu(\phi_c+\phi)^j + \frac{1}{2}e_R^2\mu^{2\epsilon}A_\mu A^\mu(\phi+\phi_c)^2$$
$$- \frac{\lambda_R}{4}\mu^\epsilon\left((\phi_c+\phi)^i(\phi_c+\phi)^i\right)^2\Big]$$

$$= \int d^dx\Big[\frac{1}{2}\partial_\mu\phi^i\partial^\mu\phi^i - \cancel{\frac{1}{2}m_R^2\phi_c^2} - \cancel{m_R^2\phi_c^i\phi^i} - \frac{1}{2}m_R^2\phi^2 + \frac{1}{2}A_\mu(\partial^2g^{\mu\nu}+\partial^\mu\partial^\nu)A_\nu$$
$$+ e_R\mu^\epsilon A_\mu\epsilon^{ij}\phi_c^i\partial^\mu\phi^j + e_R\mu^\epsilon A_\mu\epsilon^{ij}\phi^i\partial^\mu\phi^j + \frac{1}{2}e_R^2\mu^{2\epsilon}A_\mu A^\mu(\phi^2 + 2\phi^i\phi_c^i + \phi_c^2)$$
$$- \frac{\lambda_R}{4}\mu^\epsilon\left((\phi^2)^2 + 4(\phi^i\phi_c^i)^2 + \cancel{(\phi_c^2)^2} + 4\phi^2(\phi^i\phi_c^i) + \cancel{4\phi_c^2(\phi^i\phi_c^i)} + 2\phi^2\phi_c^2\right)\Big]$$

$$S[\phi_c]\Big|_{\text{all other fields}=0} = \int d^dx\left[\cancel{-\frac{1}{2}m_R^2\phi_c^2} - \cancel{\frac{\lambda_R}{4}\mu^{2\epsilon}(\phi_c^2)^2}\right]$$ ← This the (negative of the) tree-level effective potential

$$\frac{\delta S[\phi_c]}{\delta\phi^i_{c,x}}\phi^i_x = \int d^4x\left[\cancel{-m_R^2\phi_c^i\phi^i} - \cancel{\lambda_R\mu^{2\epsilon}\phi_c^2(\phi_c^i\phi^i)}\right]$$

Subtract $S[\phi_c]$ and $\frac{\delta S[\phi_c]}{\delta\phi^i_{c,x}}\phi_x$ from $S[\phi_c+\phi]$ to get S_{New}.

(cancellations shown)

$\frac{1}{2\xi}(\mathcal{F})^2$

$$S_{New}[\phi_c;\phi,A] = \int d^dx \Big[\frac{1}{2}\partial_\mu \phi^i \partial^\mu \phi^i - \frac{1}{2} m_R^2 \phi^2 - \lambda_R \mu^\epsilon (\phi^i \phi_c^i)^2 - \frac{\lambda_R}{2}\mu^\epsilon \phi^2 \phi_c^2$$
$$+ \frac{1}{2} A_\mu (\partial^2 g^{\mu\nu} + \partial^\mu \partial^\nu) A_\nu + \frac{1}{2} e_R^2 \mu^{2\epsilon} A_\mu A^\mu \phi_c^2 + e_R \mu^\epsilon A_\mu \epsilon^{ij} \phi_c^i \partial^\mu \phi^j \Big]$$
$$+ \int d^dx \Big[e_R \mu^\epsilon A_\mu \epsilon^{ij} \phi^i \partial^\mu \phi^j + \frac{1}{2} e_R^2 \mu^{2\epsilon} A_\mu A^\mu (\phi^2 + 2\phi^i \phi_c^i)$$
$$- \frac{\lambda_R}{4}\mu^\epsilon \big((\phi^2)^2 + 4\phi^2 (\phi^i \phi_c^i) \big) \Big]$$

Ignoring the interaction part (irrelevant for one-loop effective potential calculations), (dropping μ^ϵ

$$S_{New}^{(quad)}[\phi_c;\phi,A] = \int d^dx \Big[\frac{1}{2}\phi^i \Big((-\partial^2 - m_R^2 - \lambda_R \phi_c^2)\delta^{ij} - 2\lambda_R \phi_c^i \phi_c^j \Big) \phi^j$$
$$+ \frac{1}{2} A_\mu \Big((\partial^2 + e_R^2 \phi_c^2) g^{\mu\nu} - \partial^\mu \partial^\nu \Big) A_\nu + \underbrace{e_R A_\mu \epsilon^{ij} \phi_c^i \partial^\mu \phi^j}_{\phi/A \text{ mixing term — gauge fixed away.}} \Big]$$

Proceed to fix a gauge — (Lorentz covariant) R_ξ gauge.

— See gauge fixing.

Choose $\mathcal{F}^a = (\partial_\mu A^\mu - \xi e_R \epsilon^{ij} \phi_c^i \phi^j)$

$gT\langle\phi\rangle^j = e_R \epsilon^{ij} \phi_c^i$.

Then, we'll get extra terms in the action:

$$S_{GF+ghost}[A,\eta] = \int d^dx \Big[-\frac{1}{2\xi}(\partial.A)^2 - \underbrace{\frac{\xi}{2} e_R \epsilon^{ij} \phi_c^i \phi^j e_R \epsilon^{kl} \phi_c^k \phi^l}_{\text{This may add an extra term to } V_{tree}}$$
$$\underbrace{- A_\mu e_R \epsilon^{ij} \phi_c^i \partial_\mu \phi^j}_{\text{cancels mixing term}} + \partial_\mu \bar\eta \partial^\mu \eta - \xi \bar\eta \eta e_R \epsilon^{ij} \phi_c^i e_R \epsilon^{kj} \phi_c^k \underbrace{- \xi \bar\eta \eta e_R \epsilon^{ij} \phi_c^i e_R \epsilon^{kj} \phi^k}_{\text{part of the interaction}} \Big]$$

Then, using $\epsilon^{ij}\epsilon^{kl} = \delta^{ik}\delta^{jl} - \delta^{il}\delta^{jk}$, and $\epsilon^{ij}\epsilon^{kj} = +\delta^{ik}$,

$$S_{GF+Ghost}[A,\eta] = \int d^dx \Big[\frac{1}{2\xi} A_\mu \partial^2 g^{\mu\nu} A_\nu - \frac{\xi}{2} e_R^2 \phi^i (\phi_c^2 \delta^{ij} - \phi_c^i \phi_c^j) \phi^j$$
$$+ \bar\eta(-\partial^2 - \xi e_R^2 \phi_c^2)\eta - \underbrace{\xi e_R^2 \phi_c^i \bar\eta \eta \phi^i}_{\text{(interaction)}} \Big]$$

The properly gauge-fixed action (quadratic part only) is:

$$S_{\text{New}}^{(\text{Quad})}[\phi_c;\phi,A,\eta] = \int d^dx \Big[\frac{1}{2}\phi^i\Big(\big(-\partial^2 - m_R^2 - \lambda_R\phi_c^2 - \xi e_R^2\phi_c^2\big)\delta^{ij} + \big(-2\lambda_R + \xi e_R^2\big)\phi_c^i\phi_c^j\Big)\phi^j$$

$$+ \frac{1}{2}A_\mu\Big(\big(\partial^2 + e_R^2\phi_c^2\big)g^{\mu\nu} - \big(1-\tfrac{1}{\xi}\big)\partial^\mu\partial^\nu\Big)A_\nu + \bar{\eta}\big(-\partial^2 - \xi e_R^2\phi_c^2\big)\eta \Big]$$

Proceed to evaluate $V_{\text{eff}}(\phi_c) = -\frac{1}{V}\left(-i\ln Z_{\text{New}}^{(\text{Quad})}[\phi_c]\right)$. $\Rightarrow$ Diagonalize the action.

Start by expressing the action in momentum space:

$$S_{\text{New}}^{(\text{Quad})} = \int \frac{d^dp}{(2\pi)^d}\Big[\frac{1}{2}\tilde{\phi}^i\Big(\big(p^2 - m_R^2 - \lambda_R\phi_c^2 - \xi e_R^2\phi_c^2\big)\delta^{ij} + \big(-2\lambda_R + \xi e_R^2\big)\phi_c^i\phi_c^j\Big)\tilde{\phi}^j$$

$$+ \frac{1}{2}\tilde{A}_\mu\Big(\big(-p^2 + e_R^2\phi_c^2\big)g^{\mu\nu} + \big(1-\tfrac{1}{\xi}\big)p^\mu p^\nu\Big)\tilde{A}_\nu + \tilde{\bar{\eta}}\big(p^2 - \xi e_R^2\phi_c^2\big)\tilde{\eta}\Big]$$

Now split operator into "transverse" $\delta^{ij} - \frac{\phi_c^i\phi_c^j}{\phi_c^2}$, $g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}$

and "longitudinal" $\frac{\phi_c^i\phi_c^j}{\phi_c^2}$, $\frac{p^\mu p^\nu}{p^2}$ components.

$$\frac{1}{2}\tilde{\phi}^i\,\mathcal{O}^{ij}\,\tilde{\phi}^j \longrightarrow \frac{1}{2}\tilde{\phi}^i\Big[\big(p^2 - m_R^2 - \lambda_R\phi_c^2 - \xi e_R^2\phi_c^2\big)\Big(\delta^{ij} - \frac{\phi_c^i\phi_c^j}{\phi_c^2}\Big)$$

$$+ \Big(-2\lambda_R + \cancel{\xi e_R^2} + \frac{p^2 - m_R^2 - \lambda_R\phi_c^2 - \cancel{\xi\phi_c^2 e_R^2}}{\phi_c^2}\Big)\phi_c^i\phi_c^j\Big]\tilde{\phi}^j$$

$$= \frac{1}{2}\tilde{\phi}^i\Big[\big(p^2 - m_R^2 - \lambda_R\phi_c^2 - \xi e_R^2\phi_c^2\big)\Big(\delta^{ij} - \frac{\phi_c^i\phi_c^j}{\phi_c^2}\Big) + \big(p^2 - m_R^2 - 3\lambda_R\phi_c^2\big)\frac{\phi_c^i\phi_c^j}{\phi_c^2}\Big]\tilde{\phi}^j$$

and,

$$\frac{1}{2}\tilde{A}_\mu\,\mathcal{O}^{\mu\nu}\tilde{A}_\nu \longrightarrow \frac{1}{2}\tilde{A}_\mu\Big[\big(-p^2 + e_R^2\phi_c^2\big)\Big(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\Big) + \Big(\cancel{1} - \frac{1}{\xi} + \frac{\cancel{-p^2} + e_R^2\phi_c^2}{p^2}\Big)p^\mu p^\nu\Big]\tilde{A}_\nu$$

$$= \frac{1}{2}\tilde{A}_\mu\Big[\big(-p^2 + e_R^2\phi_c^2\big)\Big(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\Big) + \frac{1}{\xi}\big(-p^2 + \xi e_R^2\phi_c^2\big)\frac{p^\mu p^\nu}{p^2}\Big]\tilde{A}_\nu$$

Since $\mathrm{Tr}\left[\delta^{ij} - \frac{\phi_c^i \phi_c^j}{\phi_c^2}\right] = 1$, and $\mathrm{Tr}\left[\frac{\phi_c^i\phi_c^j}{\phi_c^2}\right] = 1$, I can find a suitable rotation that will take

$$R^{-1}_{\ell i}\left(\delta^{ij} - \frac{\phi_c^i\phi_c^j}{\phi_c^2}\right) R_{jk} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}_{\ell k}, \qquad R = \frac{1}{\sqrt{(\phi_c)^2}}\begin{pmatrix} \phi^2 & \phi_c^1 \\ -\phi_c^1 & \phi_c^2 \end{pmatrix}$$

$$R^{-1}_{\ell i}\ \frac{\phi_c^i\phi_c^j}{\phi_c^2}\ R_{jk} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

Similarly, $\mathrm{Tr}\left[g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right] = 3$ and $\mathrm{Tr}\left[\frac{p^\mu p^\nu}{p^2}\right] = 1$, so I can find another rotation (boost) matrix:

$$R^{-1}_{\rho\mu}\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right) R_{\nu\sigma} = \begin{pmatrix} 0 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \ddots \end{pmatrix}_{\rho\sigma} \quad \text{and}$$

in Dim reg.

$$R^{-1}_{\rho\mu}\left(\frac{p^\mu p^\nu}{p^2}\right) R_{\nu\sigma} = \begin{pmatrix} 1 & & & \\ & 0 & & \\ & & 0 & \\ & & & 0 \ddots \end{pmatrix}_{\rho\sigma}.$$

So that $\left(\tilde{\phi}^i_{rot} = R^{ij}\tilde{\phi}^j \ \&\ \tilde{A}^{rot}_\mu = R_\mu{}^\nu \tilde{A}_\nu\right)$

$$S^{(Quad)}_{New} = \int \frac{d^dp}{(2\pi)^d}\Bigg[\frac{1}{2}\tilde{\phi}^i_{rot.}\left(\left(p^2 - m_R^2 - \lambda_R\phi_c^2 - \xi e_R^2\phi_c^2\right)\begin{pmatrix} 1 & \\ & 0\end{pmatrix}^{ij} + \left(p^2 - m_R^2 - 3\lambda_R\phi_c^2\right)\begin{pmatrix} 0 & \\ & 1\end{pmatrix}^{ij}\right)\tilde{\phi}^j_{rot}$$

$$+ \frac{1}{2}\tilde{A}^{rot}_\mu\left[\left(-p^2 + e_R^2\phi_c^2\right)\begin{pmatrix} 0 & & & \\ & 1 & & \\ & & 1 & \\ & & & \ddots\end{pmatrix}^{\mu\nu} + \frac{1}{\xi}\left(-p^2 + \xi e_R^2\phi_c^2\right)\begin{pmatrix} 1 & & & \\ & 0 & & \\ & & 0 & \\ & & & \ddots\end{pmatrix}^{\mu\nu}\right]\tilde{A}^{rot}_\nu$$

$$+ \tilde{\bar{\eta}}\left(p^2 - \xi e_R^2\phi_c^2\right)\tilde{\eta}\Bigg]$$

The action is diagonalized — take the functional integral:

$$-i\ln Z_{New}(\phi_c) = -iV\int\frac{d^dp}{(2\pi)^d}\Bigg[\underbrace{-\frac{1}{2}\ln\left(p^2 - m_R^2 - \lambda_R\phi_c^2 - \xi e_R^2\phi_c^2\right)}_{\text{Goldstone DoF}} \underbrace{-\frac{1}{2}\ln\left(p^2 - m_R^2 - 3\lambda_R\phi_c^2\right)}_{\text{Physical Scalar DoF}}$$

$$\underbrace{-\frac{1}{2}(d-1)\ln\left(-p^2 + e_R^2\phi_c^2\right)}_{\text{Photon}} - \frac{1}{2}\ln\left(\frac{1}{\xi}\left(-p^2 + \xi e_R^2\phi_c^2\right)\right) + \underbrace{\ln\left(p^2 - \xi e_R^2\phi_c^2\right)}_{\text{Ghost}}\Bigg]$$

Factor out $\ln(-1) = i\pi$ and $-\ln(\xi)$.

$$-i \ln Z_{New} = \frac{i}{2} V \int \frac{d^d p}{(2\pi)^d} \Big[\ln\left(p^2 - m_R^2 - \lambda_R \phi_c^2 - \xi e_R^2 \phi_c^2\right) + \ln\left(p^2 - m_R^2 - 3\lambda_R \phi_c^2\right)$$

$$(d-1)\ln\left(p^2 - e_R^2\phi_c^2\right) + \underbrace{\ln\left(p^2 - \xi e_R^2 \phi_c^2\right) - 2\ln\left(p^2 - \xi e_R^2\phi_c^2\right)}_{\text{add these}} \underbrace{+ 4i\pi - \ln\xi}_{\text{Drop these - const.}} \Big]$$

$$= \ln\left(\frac{p^2 - m_R^2 - \lambda_R\phi_c^2 - \xi e_R^2 \phi_c^2}{p^2 - \xi e_R^2 \phi_c^2}\right)$$

$$= \ln\left(1 - \frac{m_R^2 + \lambda_R \phi_c^2}{p^2 - \xi e_R^2 \phi_c^2}\right)$$

So, finally

$$V_{eff}^{1\text{-loop}}(\phi_c) = -\frac{1}{V}\left(-i \ln Z_{new}\right)$$

$$= \frac{-i}{2}\int \frac{d^d p}{(2\pi)^d}\Big[\ln\big(p^2 - \overbrace{m_R^2 - 3\lambda\phi_c^2}^{m_\phi^2(\phi_c)}\big) + (d-1)\ln\big(p^2 - \overbrace{e_R^2\phi_c^2}^{m_A^2(\phi_c)}\big)$$

$$+ \ln\Big(1 - \frac{\overbrace{m_R^2 + \lambda_R\phi_c^2}^{m_G^2(\phi_c)}}{p^2 - \xi e_R^2 \phi_c^2}\Big)\Big]$$

$$= \frac{-i}{2}\int \frac{d^d p}{(2\pi)^d}\Big[\ln\left(p^2 - m_\phi^2(\phi_c)\right) + (d-1)\ln\left(p^2 - m_A^2(\phi_c)\right) + \ln\Big(1 - \frac{m_G^2(\phi_c)}{p^2 - \xi m_A^2(\phi_c)}\Big)\Big]$$

Evaluation of these integrals is straightforward:

Write $\ln\left(1 - \frac{m_G^2(\phi_c)}{p^2 - \xi m_A^2(\phi_c)}\right) = \ln\left(p^2 - m_G^2(\phi_c) - \xi m_A^2(\phi_c)\right) - \ln\left(p^2 - \xi m_A^2(\phi_c)\right)$

Then,

$$V_{eff}^{1\text{-loop}}(\phi_c) = \frac{-i}{2}\int\frac{d^d p}{(2\pi)^d}\Big[\ln\left(p^2 - m_\phi^2(\phi_c)\right) + (d-1)\ln\left(p^2 - m_A^2(\phi_c)\right) + \ln\left(p^2 - m_G^2(\phi_c) - \xi m_A^2(\phi_c)\right)$$

$$- \ln\left(p^2 - \xi m_A^2(\phi_c)\right)\Big]$$

$$= \frac{-1}{4(4\pi)^2}\Big[\left(m_\phi^2(\phi_c)\right)^2\left(\alpha_\epsilon - \ln\left(\frac{m_\phi^2(\phi_c)}{\mu^2}\right) + \frac{3}{2}\right) + 3\left(m_A^2(\phi_c)\right)^2\left(\alpha_\epsilon - \ln\left(\frac{m_A^2(\phi_c)}{\mu^2}\right) + \frac{5}{6}\right)$$

$$+ \left(m_G^2(\phi_c) + \xi m_A^2(\phi_c)\right)^2\left(\alpha_\epsilon - \ln\left(\frac{m_G^2(\phi_c) + \xi m_A^2(\phi_c)}{\mu^2}\right) + \frac{3}{2}\right) - \left(\xi m_A^2(\phi_c)\right)^2\left(\alpha_\epsilon - \ln\left(\frac{\xi m_A^2(\phi_c)}{\mu^2}\right) + \frac{3}{2}\right)$$

Proceed to renormalize (in $\overline{MS}$)

Tree level effective potential is:

$$V_{\text{eff}}^{\text{Tree}}(\phi_c) = \frac{1}{2} m_R^2 \phi_c^2 + \frac{\lambda_R}{4} (\phi_c^2)^2$$

One-loop divergent part:

Now, $m_\phi^2(\phi_c) = m_R^2 + 3\lambda_R \phi_c^2$

$$\Rightarrow (m_\phi^2(\phi_c))^2 = \underline{m_R^4 + 6\lambda_R m_R^2 \phi_c^2 + 9\lambda_R^2 (\phi_c^2)^2}$$

$$m_A^2(\phi_c) = e_R^2 \phi_c^2 \Rightarrow 3(m_A^2(\phi_c))^2 = \underline{3 e_R^4 \phi_c^4}$$

$$m_G^2(\phi_c) + \xi m_A^2(\phi_c) = m_R^2 + \lambda_R \phi_c^2 + \xi e_R^2 \phi_c^2$$

$$\Rightarrow (m_G^2(\phi_c) + \xi m_A^2(\phi_c))^2 = \underline{m_R^4 + 2(m_R^2 \lambda_R + \xi m_R^2 e_R^2)\phi_c^2 + (2\xi \lambda_R e_R^2 + \lambda_R^2 + \xi^2 e_R^4)\phi}$$

and $\quad -(\xi m_A(\phi_c))^2 = -\xi^2 e_R^4 \phi_c^4$

Add, and group by powers of ϕ_c:

The divergent polynomial is:

$$= \frac{-1}{4(4\pi)^2}\left[2 m_R^4 + 2(4\lambda_R + \xi e_R^2) m_R^2 \phi_c^2 + (10\lambda_R^2 + 3e_R^4 + 2\xi\lambda e_R^2)\phi^4\right] \frac{1}{\epsilon} - \gamma_E$$

Hence, $\quad \delta_{m^2} = \frac{1}{(4\pi)^2}(4\lambda_R + \xi e_R^2) m_R^2 \left(\frac{1}{\epsilon} - \gamma_E + \ln 4\pi\right)$

$$\delta_{\phi^4} = \frac{1}{(4\pi)^2}\left(10\lambda_R^2 + 3e_R^4 + 2\xi\lambda_R e_R^2\right)\left(\frac{1}{\epsilon} - \gamma_E + \ln 4\pi\right)$$

Separate computation required

$$\left\{\begin{array}{l} \delta_A = \dfrac{-e_R^2}{3(4\pi)^2}\left(\dfrac{1}{\epsilon} - \gamma_E + \ln 4\pi\right) \quad ✓ \\ \delta_\phi = (3+\xi)\dfrac{e_R^2}{(4\pi)^2} \end{array}\right.$$