Gauge Dependence of the Effective Potential (Summary)

To understand the dependence on ξ of V_{eff} requires the full machinery of the BRST symmetry.

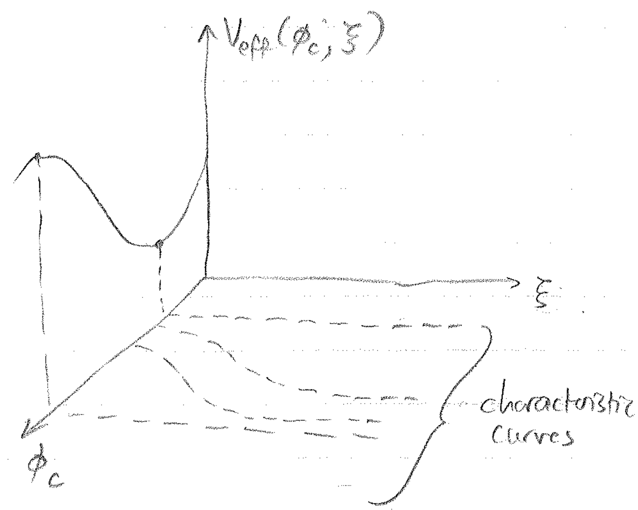
Following the same logic to derive the Ward Identities for abelian gauge theories, or the Taylor-Slavnov Identities for non-abelian gauge theories, one can derive analogous Nielsen Identities for effective potentials. — long and complicated argument.

The Nielsen Identity tells you how the effective potential depends on the gauge parameter, ξ.

$$\frac{\partial V_{eff}}{\partial \xi} = -C(\phi_c, \xi)\,\frac{\partial V_{eff}}{\partial \phi_c}$$

Note: when $\frac{\partial V_{eff}}{\partial \phi_c} = 0$, we have $\frac{\partial V_{eff}}{\partial \xi} = 0$. At extrema, the effective potential is gauge independent.

Also, can be viewed as a PDE: if V_{eff} is known for one choice of ξ, one can solve the PDE to find out what V_{eff} looks like for other choices of ξ.



Method of characteristics:

Along the curves in ϕ_c–ξ plane defined by $\frac{d\sigma}{d\xi} = C(\sigma, \xi)$,

The effective potential is gauge indep. everywhere:

$$\left.\frac{dV_{eff}}{d\xi}\right|_{\sigma:\ \frac{d\sigma}{d\xi} = C(\sigma,\xi)} = 0.$$

However, these statements are untrue at any finite order in perturbation theory.

Write
$$V_{eff} = V^0 + \hbar V^1 + \hbar^2 V^2 + \ldots$$
$$C = c_0 + \hbar c_1 + \hbar^2 c_2 + \ldots$$

$$\frac{\partial V^0}{\partial \xi} + \hbar \frac{\partial V^1}{\partial \xi} + \ldots = -\left(c_0 + \hbar c_1 + \ldots\right)\left(\frac{\partial V^0}{\partial \phi_c} + \hbar \frac{dV^1}{d\phi_c} + \ldots\right)$$
$$= -c_0 \frac{\partial V^0}{\partial \phi_c} - \hbar\left(c_0 \frac{\partial V^1}{\partial \phi_c} + c_1 \frac{\partial V^0}{\partial \phi_c}\right) + \ldots$$

The tree-level effective potential is gauge independent:
$$\frac{\partial V^0}{\partial \xi} = 0 \quad \Rightarrow \quad c_0 = 0.$$

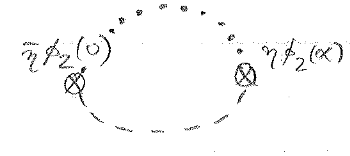
Hence at one-loop,
$$\frac{\partial V^1}{\partial \xi} = -c_1 \frac{\partial V^0}{\partial \phi_c}$$

$\Rightarrow$ the one-loop effective potential is gauge invariant only where the <u>tree-level</u> effective potential is extremized!

<u>nb</u>! for scalar QED: ? sign?

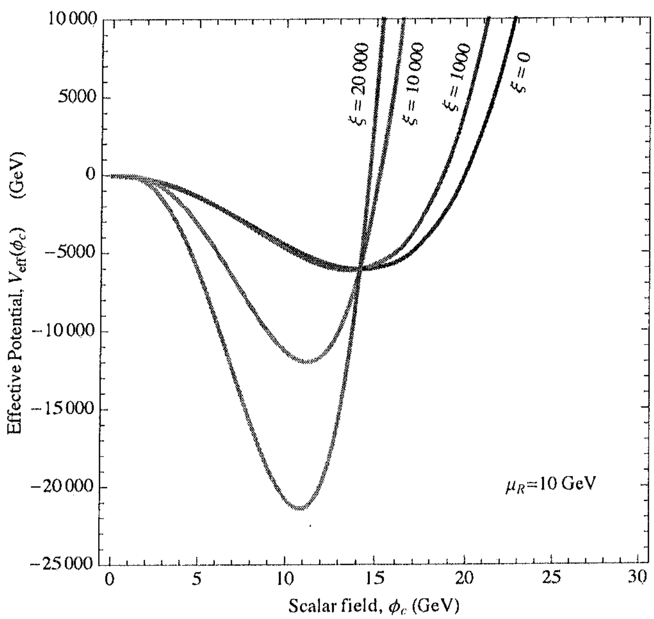
$$C(\phi_c, \xi) = -\int d^4x \, \langle 0 | T \, \frac{1}{2\xi} \underbrace{\bar{\eta}\left(\partial^\mu \hat{A}_\mu - \xi e \phi_c \hat{\phi}_2\right)}_{\text{fields evaluated at } x=0.} \left(-e\eta(x)\hat{\phi}_2(x)\right) | 0 \rangle$$

at one loop:



Gauge dependence of the Effective Potential

Gauge dependence of the Coleman-Weinberg effective potential of **Scalar QED** at zero temperature. Large changes in the gauge fixing parameter, ξ induces large changes in the effective potential.



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