Gauge Dependence of the Sphaleron Rate

Nielsen's identities are applicable if:

1. BRST continues to hold at finite temperature
   – it does provided FP ghost fields satisfy periodic boundary conditions over imaginary time interval.

$$\eta(\vec{x},\tau) = +\eta(\vec{x},\tau+\beta).$$

2. The Legendre transformation of the partition function, $Z(T)$, that gives the Euclideanized thermal effective action, $\Gamma_E^{(\beta)}$, is the combination $E_{sp}(T)/T$ that goes in the exponent of the sphaleron rate law:

– It is: $$\Gamma_E^{(\beta)}[\phi_{sp}, A^\mu_{sp}] = \int_0^{1/T} d\tau \int d^3x\, \mathcal{L}_E^{\text{eff}}[\phi_{sp}, A^\mu_{sp}; T]$$

ϕ_{sp}, A^μ_{sp} obtained by requiring

$$\left.\frac{\delta\Gamma_E^{(\beta)}}{\delta\phi}\right|_{\phi=\phi_{sp}} = \left.\frac{\delta\Gamma_E^{(\beta)}}{\delta A_\mu}\right|_{A^\mu=A^\mu_{sp}} = 0$$

simultaneously with $\dot{\phi}_{sp} = \dot{A}^\mu_{sp} = 0$ (req: $A^0=0$ gauge)

Then $$\Gamma_E^{(\beta)}[\phi_{sp}, A_{sp}] = \frac{1}{T}\underbrace{\int d^3x\, \mathcal{L}_E^{\text{eff}}[\phi_{sp}, A^\mu_{sp}; T]}_{E_{sp}(T)}$$

$$= \frac{E_{sp}(T)}{T} \quad \checkmark$$ – see kapusta

So, the gauge dependence of the Energy functional is (Nielsen id):

$$\frac{\partial\Gamma_E^{(\beta)}}{\partial\xi} = \int d^4x \left[\left(\int d^4y \frac{\delta^2\Gamma_E^{(\beta)}}{\delta K_{GF}(y)\,\delta(k_\phi^c)_i(x)}\right)\frac{\delta\Gamma_E^{(\beta)}}{\delta\phi_i(x)} + \left(i D^{\mu ab}_{(x)} \int d^4y \frac{\delta^2\Gamma_E^{(\beta)}}{\delta K_{GF}(y)\,\delta(k_\eta)^b(x)}\right)\frac{\delta\Gamma_E^{(\beta)}}{\delta A^{\mu a}(x)}\right]_{\substack{\phi_{sp}\\ A^\mu_{sp}}}$$

But since these vanish when evaluated at the stationary point, ϕ_{sp}, A^μ_{sp}, we have:

$$\boxed{\frac{\partial\Gamma_E^{(\beta)}}{\partial\xi} = \frac{\partial}{\partial\xi}\left(\frac{E_{sp}(T)}{T}\right) = 0}$$ (provided T doesn't depend on ξ)