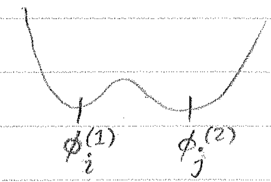
Gauge Dependence of T_c and ϕ_{crit}

Denote finite temperature effective potential (free energy) by $V_{eff}(\phi_i, T; \xi)$ ← Gauge param.

Critical temperature defined by: (nec. but not suf. cond — need $\frac{d^2V}{d\phi^2} > 0$)



① $V_{eff}(\phi_i^{(1)}, T_c; \xi) - V_{eff}(\phi_i^{(2)}, T_c; \xi) = 0$

② $\frac{\partial V_{eff}}{\partial \phi_i}(\phi_i^{(1)}, T_c; \xi) = \frac{\partial V_{eff}}{\partial \phi_i}(\phi_i^{(2)}, T_c; \xi) = 0$

$\phi_i^{(1)}$, $\phi_i^{(2)}$ and T_c are found by simultaneously solving / inverting the above eqns.

$\Rightarrow$ obtain $\phi_1(\xi, \underbrace{\ldots}_{\text{parameters of theory}})$ and $\phi_2(\xi, \ldots)$ and $T_c(\xi, \ldots)$

Differentiate ① with respect to ξ:

$$0 = \left(\frac{\partial V_{eff}}{\partial \phi_i}\frac{\partial \phi_i^{(1)}}{\partial \xi} + \frac{\partial V_{eff}}{\partial T}\frac{\partial T_c}{\partial \xi} + \frac{\partial V_{eff}}{\partial \xi}\right)_{\phi=\phi^{(1)}} - \left(\frac{\partial V_{eff}}{\partial \phi_i}\frac{\partial \phi_i^{(2)}}{\partial \xi} + \frac{\partial V_{eff}}{\partial T}\frac{\partial T_c}{\partial \xi} + \frac{\partial V_{eff}}{\partial \xi}\right)_{\phi=\phi^{(2)}}$$

At $\phi = \phi^{(1)}$ and $\phi = \phi^{(2)}$,

we have $\frac{\partial V_{eff}}{\partial \phi_i} = 0$. Then by N. iden: $\frac{\partial V_{eff}}{\partial \xi} = 0$

$$0 = \underbrace{\left(\left.\frac{\partial V_{eff}}{\partial T}\right|_{\phi^{(1)}} - \left.\frac{\partial V_{eff}}{\partial T}\right|_{\phi^{(2)}}\right)}_{\text{In general doesn't vanish}} \frac{\partial T_c}{\partial \xi}$$

$$\Rightarrow \boxed{\frac{\partial T_c}{\partial \xi} = 0}$$

Hence, critical temperatures are indep. of gauge parameter, ξ.

Gauge dep. of order parameter, ϕ_i :

Differentiate ② wrt ξ :

$$\left.\frac{\partial^2 V_{eff}}{\partial\phi_i\partial\phi_j}\right| \frac{\partial\phi_j^{(1,2)}}{\partial\xi} + \left.\frac{\partial^2 V_{eff}}{\partial T\partial\phi_i}\right| \underbrace{\frac{\partial T_c}{\partial\xi}}_{\substack{=0 \\ \text{(established)}}} + \underbrace{\left.\frac{\partial^2 V_{eff}}{\partial\xi\,\partial\phi_i}\right|_{\phi_i^{(1,2)}}}_{\substack{\text{Use Nielsen} \\ \text{Identity:}}} = 0$$

$$\frac{\partial V_{eff}}{\partial\xi} = -C_j(\phi,\xi)\frac{\partial V_{eff}}{\partial\phi_j}$$

Diff:

$$\frac{\partial^2 V_{eff}}{\partial\xi\partial\phi_i} = -\frac{\partial C_j}{\partial\phi_i}\frac{\partial V_{eff}}{\partial\phi_j} - C_j(\phi,\xi)\frac{\partial^2 V_{eff}}{\partial\phi_i\partial\phi_j}$$

when evaluated at minimum, this vanishes.

So,
$$\left(\frac{\partial\phi_j^{(1,2)}}{\partial\xi} - C_j(\phi^{(1,2)},\xi)\right)\underbrace{\left.\frac{\partial^2 V_{eff}}{\partial\phi_i\partial\phi_j}\right|_{\phi^{(1,2)}}}_{} = 0$$

This is the mass-matrix
(does not vanish in general)

$$\Rightarrow \boxed{\frac{\partial\phi_j^{(1,2)}}{\partial\xi} = C_j(\phi,\xi)}$$
, as expected, since they follow characteristic curves.