V_{eff} from 3D effective theory:

4D theory:

$$S = \int_0^\beta d\tau \int d^3x \left[\frac{1}{2} D_\mu \Phi D_\mu \Phi + V(\Phi) + \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} \right]$$

→ Dimensional reduction

no cubic

$$= \beta \int d^3x \left[\frac{1}{4} F^a_{ij} F^a_{ij} + \frac{1}{2} (D_i A_0)^a (D_i A_0)^a + (\vec{D}\,\Phi)^\dagger_i (\vec{D}\,\Phi)_i + V(\Phi, T; \xi) \right.$$
$$\left. + \frac{1}{2} \Pi^{ab}(0) A_0^a A_0^b + \mathcal{O}(A_0^4) + \dots \right]$$

Drop all terms in V^{4D}_{eff} that are gauge dependent — keep T^2 terms only. Proceed to compute effective potential.

Shift: $\Phi \to \bar{\phi} + \phi$

Gauge-fixing: $\mathcal{L}_\xi = \frac{1}{2\xi} \left(\vec{\partial} \cdot \vec{A}^a + \xi \phi_i (g T^a \bar{\phi})_i \right)^2$

Result:

$$= \beta \int d^3x \left[\frac{1}{2} A^a_\mu \left(-\vec{\partial}^2 \delta_{\mu\nu} + \left(1 - \frac{1}{\xi}\right) \partial_i \partial_j + m_A^2(\bar{\phi})^{ab} \delta_{\mu\nu} + \Pi^{ab}(0) \frac{n_\mu n_\nu}{n^2} \right) A^b_\nu \right.$$
$$+ \frac{1}{2} \phi_i \left(-\vec{\partial}^2 \delta_{ij} + M^2_{ij}(\bar{\phi}) + \xi m_A^2(\bar{\phi})_{ij} + \Sigma_{ij}(0) \right) \phi_j$$
$$\left. + \eta^{\dagger a} \left(-\vec{\partial}^2 \delta^{ab} + \xi m_A^2(\bar{\phi})^{ab} \right) \eta^b \right]$$

see next page for integration steps.

$$V^{\text{1-loop}}_{\text{eff}} = \frac{1}{2\beta} \times \frac{-1}{6\pi} \sum_{\text{eval}} \left[2\left(m_A^2(\phi)\right)^{3/2} + \left(\xi m_A^2(\phi)\right)^{3/2} + \left(m_A^2(\phi) + \Pi(0)\right)^{3/2} \right.$$
$$\left. + \left(M^2(\phi) + \xi m_A^2(\phi) + \Sigma(0)\right)^{3/2} - 2\left(\xi m_A^2(\phi)\right)^{3/2} \right]$$

$$\Gamma_E[A] = \int d^3x \; \frac{1}{2} A_\mu \left(-\partial_E^2 g^{\mu\nu} + \left(1-\frac{1}{\xi}\right)\partial^i\partial^j + m_A^2\, g^{\mu\nu} + \Pi_B(0)\, \frac{n^\mu n^\nu}{n^2} \right) A_\nu$$

To momentum space

$$\int \frac{d^3p}{(2\pi)^3}\, \frac{1}{2} \tilde{A}_\mu \left(\vec{p}^{\,2} g^{\mu\nu} - \left(1-\frac{1}{\xi}\right) p^i p^j + m_A^2\, g^{\mu\nu} + \Pi_B(0)\, \frac{n^\mu n^\nu}{n^2} \right) A_\nu$$

$$= \int \frac{d^3p}{(2\pi)^3}\, \frac{1}{2} \tilde{A}_\mu \Big(\vec{p}^{\,2} g^{\mu\nu} - p^2 \underbrace{\left(1-\frac{1}{\xi}\right)\frac{p^i p^j}{\vec{p}^{\,2}}} - m_A^2 \frac{p^i p^j}{p^2} + \underbrace{m_A^2 \frac{p^i p^j}{\vec{p}^{\,2}}} + m_A^2 g^{\mu\nu}$$
$$+ \Pi_B(0)\, \frac{n^\mu n^\nu}{n^2} \Big) A_\nu$$

$$= \int \frac{d^3p}{(2\pi)^3}\, \frac{1}{2} \tilde{A}_\mu \left[(\vec{p}^{\,2} + m_A^2)\left(g^{\mu\nu} - \frac{p^i p^j}{\vec{p}^{\,2}} \right) + \left(\frac{\vec{p}^{\,2}}{\xi} + m_A^2 \right) \frac{p^i p^j}{\vec{p}^{\,2}} + \Pi_B(0)\, \frac{n^\mu n^\nu}{n^2} \right] A_\nu$$

$$g^{\mu\nu} = \frac{n^\mu n^\nu}{n^2} + \delta^{ij}$$

$$= \int \frac{d^3p}{(2\pi)^3}\, \frac{1}{2} \tilde{A}_\mu \left[(\vec{p}^{\,2} + m_A^2)\left(\delta^{ij} - \frac{p^i p^j}{\vec{p}^{\,2}} \right) + \left(\frac{\vec{p}^{\,2}}{\xi} + m_A^2 \right) \frac{p^i p^j}{p^2} + \left(\vec{p}^{\,2} - m_A^2 + \Pi_B(0) \right) \frac{n^\mu n^\nu}{n^2} \right]$$

$$= \int \frac{d^3p}{(2\pi)^3}\, \frac{1}{2} \tilde{A}_i \left[(\vec{p}^{\,2} + m_A^2)\left(\delta^{ij} - \frac{p^i p^j}{\vec{p}^{\,2}} \right) + \left(\frac{\vec{p}^{\,2}}{\xi} + m_A^2 \right) \frac{p^i p^j}{\vec{p}^{\,2}} \right] \tilde{A}_j + \frac{1}{2} \tilde{A}_0 \left(\vec{p}^{\,2} - m_A^2 + \Pi_B(0) \right) \tilde{A}_0$$

Perform functional integral:

$$V_{eff} = \frac{1}{2\beta} \int \frac{d^3p}{(2\pi)^3} \Big[\underbrace{(3-1)}_{2} \operatorname{Tr} \ln\left(\vec{p}^{\,2} + m_A^2\right) + \operatorname{Tr} \ln\left(\vec{p}^{\,2} + \xi m_A^2\right) + \operatorname{Tr} \ln\left(\vec{p}^{\,2} + m_A^2 + \Pi_B(0)\right) \Big]$$

$$= \frac{1}{2\beta} \left[\frac{-1}{6\pi}\, 2\left(m_A^2(\phi) \right)^{3/2} - \frac{1}{6\pi} \left(\xi m_A^2 \right)^{3/2} - \frac{1}{6\pi} \left(m_A^2 + \Pi_B(0) \right)^{3/2} \right]$$