Multivariable Tunneling

Following the same steps as before, start with classical action:

$$S = \int dt \left[\frac{1}{2} m \left(\frac{d\vec{x}}{dt} \right)^2 - V(\vec{x}) \right],$$

Take m the same for each DoF - since ultimately, we'll move to field th.

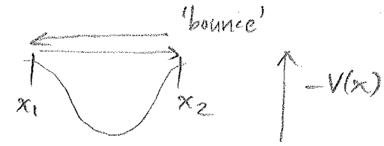
and construct the associated Euclideanized action $(t \to -i\tau)$

$$S_E = \int d\tau \left[\frac{1}{2} m \left(\frac{d\vec{x}}{d\tau} \right) + V(\vec{x}) \right]$$

Extremize action, $m \dfrac{d^2\vec{x}}{d\tau^2} = \dfrac{dV}{d\vec{x}}$, and find bounce solutions, subject to:

$$\vec{x}(-\infty) = \vec{x}_1, \quad \left.\frac{d\vec{x}}{d\tau}\right|_{\vec{x}=\vec{x}_1} = \text{"}0\text{"}$$

$$\vec{x}(\tau=0) = \vec{x}_2 \quad \left.\frac{d\vec{x}}{d\tau}\right|_{\vec{x}=\vec{x}_2} = 0 \quad \text{"turning point"}$$



The semiclassical tunneling exponential is (amplitude of tunneling)

$$\boxed{T \sim \exp\left[-\int_{-\infty}^{+\infty} d\tau \left[\frac{1}{2} m \left(\frac{d\vec{x}_B}{d\tau} \right)^2 - V(\vec{x}_B) \right] \right]}$$

(if there are multiple bounce solutions, then choose the one for which the euclideanized action, $S_E[x_B]$, is least).

Can exist because initial condition $\left.\dfrac{d\vec{x}}{d\tau}\right|_{\vec{x}=\vec{x}_1} = \text{"}0\text{"}$ really means a small push in some direction, $\epsilon \hat{x}$, giving rise to a family of paths that lead to the same bounce point, $\vec{x}_2$.