SU(2) Yang-Mills theory in 4D — review

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^{a} F^{\mu\nu a},$$

where $F_{\mu\nu}^{a} = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - \underset{\substack{\uparrow \\ \text{coupling constant}}}{g} \epsilon^{abc} A_\mu^b A_\nu^c$ is the field-strength tensor.

By construction, theory is invariant under:

1. Poincaré group (translations, Lorentz boosts, rotations)
2. Internal SU(2) gauge symmetry (redundancy)

$$A_\mu^a \longrightarrow A_\mu^a - \frac{1}{g} \partial_\mu \alpha^a(x) - \epsilon^{abc} \alpha^b(x) A_\mu^c + \mathcal{O}(\alpha^2)$$

space-time dependent parameter.

Often, it pays to express $A_\mu^a(x)$ as a matrix-valued field:

Write $A_\mu \equiv A_\mu^a T^a \leftarrow T^a \equiv \frac{\sigma^a}{2}$ = generators of SU(2) in fundamental representation

and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$
$\equiv F_{\mu\nu}^a T^a$

So that now the Lagrangian takes the form:

$$\mathcal{L} = -\frac{1}{2} \mathrm{Tr}\left[F_{\mu\nu} F^{\mu\nu}\right],$$

and the theory is invariant (rather, redundant) under

$$A_\mu \longrightarrow U(x) A_\mu U^\dagger(x) - \frac{i}{g} U(x) \partial_\mu U^\dagger(x),$$

with $U(x) = e^{i\alpha^a(x) T^a} \in SU(2)$ at each space-time point.

There are two additional accidental symmetries:

3. Scale transformations: $x^\mu \to \lambda x^\mu$
4. Special conformal transformations: $x^\mu \longrightarrow \dfrac{x^\mu + b^\mu x^2}{1 + 2 b\cdot x + b^2 x^2}$