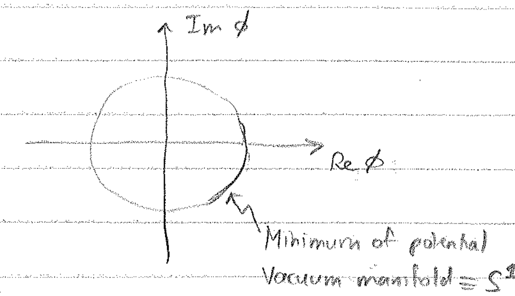
Scalar Vortex (2+1)-D



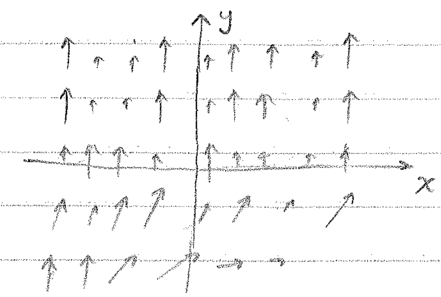
Theory: One complex scalar field ϕ

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - \lambda(\phi^*\phi - v^2)^2$$

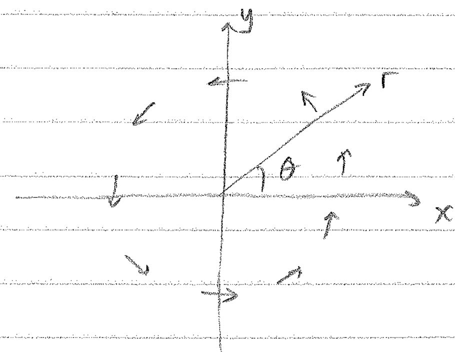
$\phi = [\text{Energy}]^{1/2}$, classical vacuua at

$\lambda = [\text{Energy}]^1$ $\qquad |\langle\phi\rangle| = v$

Schematic of field config. at fixed t:



(no winding)



$n=1$ vortex

Parametrize field config in complex polar form:

$\phi(x) = \rho(x)\, e^{i\alpha(x)}$ $\qquad$ vacuum at $\rho(x) = v$.

Also move to polar coordinates for space:

$$\vec{x} = (r\cos\theta,\ r\sin\theta)$$

Topological soliton (vortex) of the form:

$$\phi(x) = v\, e^{i\alpha(x)}$$

with $\alpha(\theta + 2\pi) = \alpha(\theta) + 2\pi n$

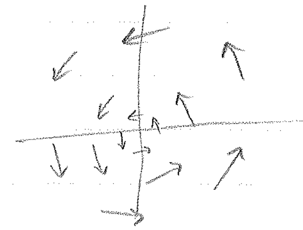
"winding number"

Winding number:

$$n = \frac{i}{2\pi}\int_0^{2\pi} d\phi \ U \partial_\phi U^\dagger \quad , \quad U = e^{in\phi}$$

(example)

Vortex of the form $\phi_v(x) = v\, U(\phi)$



Ansatz: $\phi_v(r, \theta) = v f(r)\, e^{in\theta}$

We would like for the solution to approach the true minimum at infinity $\phi_v(r \to \infty, \forall \theta) = v$. Hence $f(r = \infty) \to 1$.

To avoid a cusp at $r = 0$, $f(r=0) = 0$.

<u>Minimize</u>: $E = \int d^2x \left[\underbrace{|\dot{\phi}|^2}_{=0} + |\nabla \phi|^2 + V(\phi) \right]$

But $$\vec{\nabla}\phi = \left(\vec{e}_r \frac{\partial}{\partial r} + \vec{e}_\theta \frac{1}{r} \frac{\partial}{\partial \theta} \right) \phi_v(r, \theta)$$

$$= v\left(\vec{e}_r f'(r)\, e^{in\theta} + \vec{e}_\theta \frac{1}{r} f(r)(in)\, e^{in\theta} \right)$$

$$\vec{\nabla}\phi^\dagger = v\left(\vec{e}_r f'(r)\, e^{-in\phi} + \vec{e}_\theta \frac{1}{r} f(r)(-in)\, e^{-in\theta} \right)$$

So: $$|\nabla \phi|^2 = v^2 \left(f'(r)^2 + \frac{n^2}{r^2} f^2(r) \right)$$

The problem is $E = \int d^2x \, \frac{n^2}{r^2} f^2(r) \sim \int dr \, \frac{1}{r} \overbrace{f^2(r)}^{\text{approaches } 1} \to \int^\infty \frac{1}{r} = \infty$.

is divergent.

<u>Remedy</u>: Introduce gauge fields.

Then $|\vec{D}\phi|^2 \to |(\vec{\nabla} + ie\vec{A})\phi|^2$

choose this to cancel divergent part.

Now, at infinity $\phi_v(r, \theta) \to v\, e^{in\theta}$ is just a gauge transform of $\theta = 0$.