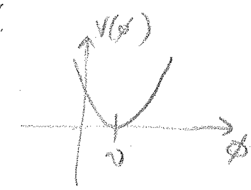
Non-topological solitons (e.g. Q-balls)

Non-topological solitons - stable by virtue of conservation of charge.

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi) + \partial_\mu \chi^* \partial^\mu \chi - g^2 \phi^2 \chi^* \chi$$

$$\uparrow$$

$$V(\phi) = \frac{m_\phi^2}{2}(\phi - v)^2$$



$m_\phi > 0, \quad v > 0, \quad h > 0.$

In 1+1 dimensional spacetime.

Invariance under global $U(1)$:

$\phi(x) \to \phi(x)$.

$\chi(x) \to e^{i\alpha}\chi(x)$ $\qquad \delta\chi = i\alpha\,\chi(x) \qquad \delta\chi^\dagger = -i\alpha\,\chi^\dagger(x)$

$\qquad \pi_\mu = \partial_\mu \chi^\dagger \qquad \pi_\mu^\dagger = \partial_\mu \chi$

Noether current:

$$j_\mu = -i\left(\partial_\mu \chi^\dagger \chi - \partial_\mu \chi \chi^\dagger\right)$$

$$Q = \int dx \left(-i\dot{\chi}^\dagger \chi + i\chi^\dagger \dot{\chi}\right)$$

Energy functional

$$E = \int dx \left[\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\left(\frac{\partial \phi}{\partial x}\right)^2 + \frac{m^2}{2}(\phi - v)^2 + |\dot{\chi}|^2 + \left|\frac{\partial \chi}{\partial x}\right|^2 + g^2 \phi^2 |\chi|^2 \right]$$

Trivial minimum: $\phi = v$, $\chi = 0$.

fluctuations: scalar quanta of masses gv and m_ϕ

local

Since charge is conserved, could we have field configurations of a given charge? By virtue of charge conservation, the object would be stable.
— Q-ball.

If charge is non-zero, fields must depend on time. $Q = \int dx\,(-i\dot{\chi}^\dagger \chi + i\chi^\dagger \dot{\chi})$

Stationary condition:

$$\frac{\partial \mathcal{L}_E}{\partial \phi} - \partial_\mu \frac{\partial \mathcal{L}_E}{\partial(\partial_\mu \phi)} = 0 \qquad \frac{\partial \mathcal{L}_E}{\partial \chi} - \partial_\mu \frac{\partial \mathcal{L}_E}{\partial(\partial_\mu \chi)} = 0$$

$$m^2(\phi - v) - \partial_\mu(\partial_\mu \phi) = 0. \qquad g^2 \phi^2 \chi^* - \partial_\mu(\partial_\mu \chi^*) = 0$$

Ok, to show there are stable local field configurations, must show energy of non-local fluctuations > local configuration:

Consider small fluctuations of $\chi(x)$ above vacuum.

Linearized theory: $E = \int dk\, \omega_k (a_k^* a_k + b_k^* b_k)$ $\quad \omega_k = \sqrt{m_\chi^2 + k^2}$

$$Q = \int dk\, (a_k^* a_k - b_k^* b_k)$$

Consider $a_k = a\,\delta(k) \Rightarrow E = m_\chi(a^2 + b^2)\,\delta(0)$

$b_k = b\,\delta(k) \Rightarrow Q = (a^2 - b^2)\,\delta(0)$

$$Q + b^2 \delta(0) = a^2 \delta(0)$$

$$E = m_\chi(Q + b^2\delta(0)) + m b^2 \delta(0)$$

$$E = m_\chi Q + 2 m_\chi b^2 \delta(0)$$

$$E > m_\chi Q$$