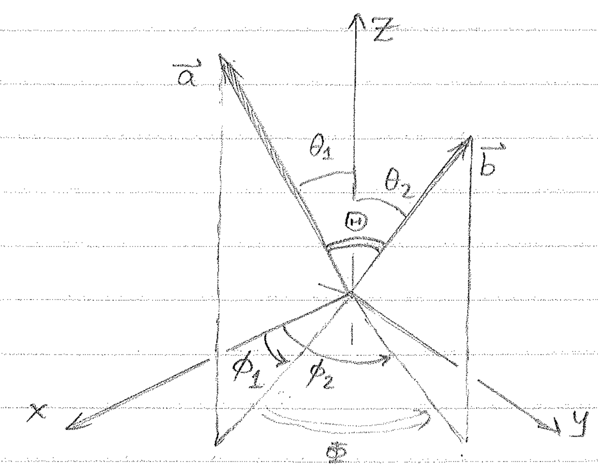
Spherical law of Cosines



Angle between two vectors $\vec{a}$ & $\vec{b}$:

$$\boxed{\begin{aligned}\cos\Theta &= \cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2\cos(\phi_1-\phi_2)\\ \Phi &= \phi_2-\phi_1\end{aligned}}$$

$$= \cos(\theta_1-\theta_2) - 2\sin\theta_1\sin\theta_2\sin^2\left(\frac{\Phi}{2}\right)$$

Proof:

Put $\vec{a} = |\vec{a}|(\sin\theta_1\cos\phi_1,\ \sin\theta_1\sin\phi_1,\ \cos\theta_1)$

$\vec{b} = |\vec{b}|(\sin\theta_2\cos\phi_2,\ \sin\theta_2\sin\phi_2,\ \cos\theta_2)$

Then

$$\cos\Theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$$

$$= \frac{1}{|\vec{a}||\vec{b}|}|\vec{a}||\vec{b}|\left(\sin\theta_1\sin\theta_2\left(\underbrace{\cos\phi_1\cos\phi_2+\sin\phi_1\sin\phi_2}\right)+\cos\theta_1\cos\theta_2\right)$$

$$= \sin\theta_1\sin\theta_2\cos(\phi_1-\phi_2) + \cos\theta_1\cos\theta_2$$

and by inspection, azimuthal angle of $\vec{b}$ relative to $\vec{a}$ is:

$$\Phi = \phi_2 - \phi_1$$