

$E-q^2$ Scattering Plane - kinematics
(Mandelstam plane for non-relativistic scattering)

$E = \frac{k^2}{2m} \hbar^2$ Total C.O.M energy $= E_1 + E_2$

$\vec{q} = \frac{1}{2}(\vec{k}' - \vec{k})$ Momentum transfer with $|\vec{k}| = |\vec{k}'| = k$ for elastic scattering
(fixed by energy conservation)

$\vec{q}' = \frac{1}{2}(\vec{k}' + \vec{k})$ Exchange-momentum transfer

Physical region:

For fixed E , $q^2 = \hbar^2(\vec{k}' - \vec{k})^2$
 $= \hbar^2(k'^2 + k^2 - 2\vec{k}' \cdot \vec{k})$
 $= 2\hbar^2 k^2(1 - \cos\theta)$
 $= 4mE(1 - \cos\theta) \equiv 8mE \sin^2(\theta/2) \Rightarrow |q| = 2\hbar|k| \sin \theta/2$

physical range of θ : $0 \leq \theta \leq \pi$

physical range of $\cos\theta$: $-1 \leq \cos\theta \leq 1$

$\Rightarrow 0 \leq q^2 \leq 8mE$

