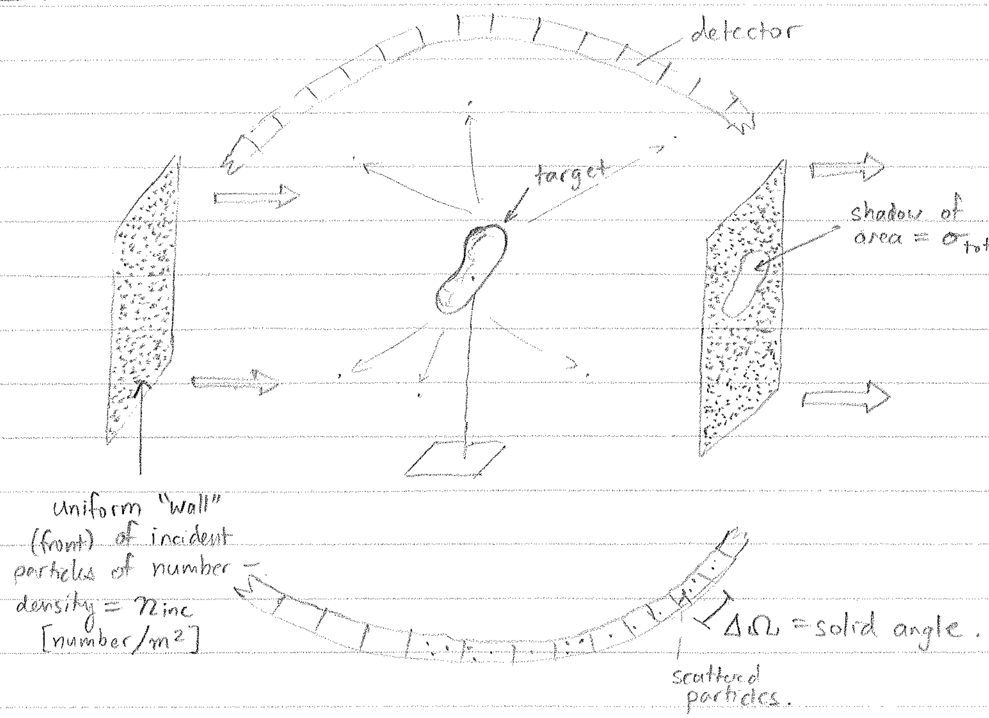
Classical scattering cross section

Setup:



Could determine total cross section by area of shadow cast by target. For tiny targets (eg. particles) this is unrealistic.

Instead, count up the total number of scattered particles, N_{scatt}.

$$\begin{pmatrix}\text{total number of}\\ \text{scattered particles}\end{pmatrix} = \begin{pmatrix}\text{number density,}\\ \text{per unit area, of}\\ \text{incident particles}\end{pmatrix}\begin{pmatrix}\text{cross section}\end{pmatrix}$$

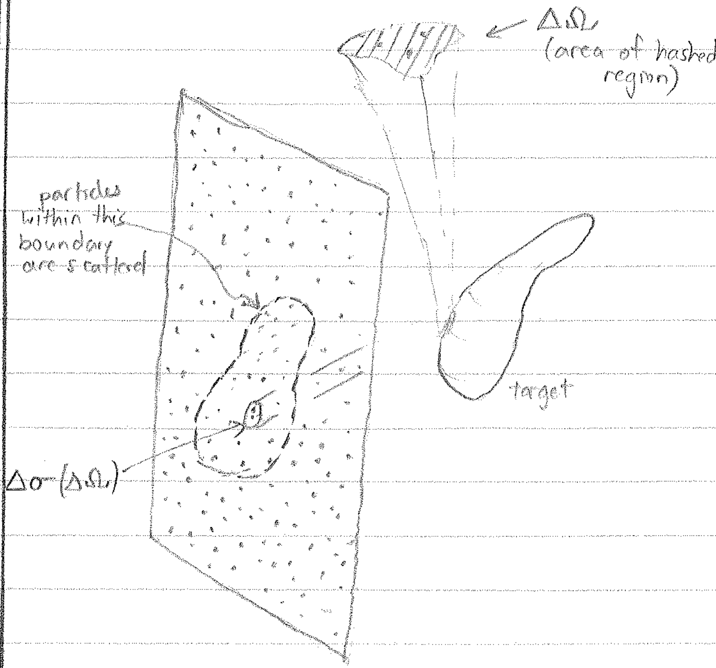
$$\underset{\substack{\uparrow\\ \text{measured by}\\ \text{experimenter}}}{N_{scatt}} = \underset{\substack{\uparrow\\ \text{set by}\\ \text{experimenter}}}{n_{inc}} \times \sigma_{tot}$$

$$\Rightarrow \boxed{\sigma_{tot} = \frac{1}{n_{inc}} N_{scatt}.}$$

Question: do we need an incident wall of particles infinite in area?

A: Not necessary. Just need it to be sufficiently larger than target.

In fact, we can learn more about the <u>shape</u> of the target by looking at how the scattered particles are distributed over the various angles.



Consider a small disk of area $\Delta\sigma$ of incident particles, and follow these particles as they get scattered into solid angle $\Delta\Omega$

$$\begin{pmatrix}\text{number of scattered}\\ \text{particles into solid}\\ \text{angle } \Delta\Omega\end{pmatrix} = \begin{pmatrix}\text{number density,}\\ \text{per unit area, of}\\ \text{incident particles}\end{pmatrix}\begin{pmatrix}\text{area of small disk } \Delta\sigma\\ \text{through which incident particles}\\ \text{would scatter into solid angle } \Delta\Omega\end{pmatrix}$$

$$\underset{\substack{\uparrow\\ \text{measured by}\\ \text{experimenter}}}{N(\Delta\Omega)} = \underset{\substack{\uparrow\\ \text{set by}\\ \text{experimenter}}}{n_{inc}} \times \Delta\sigma \qquad \circledast$$

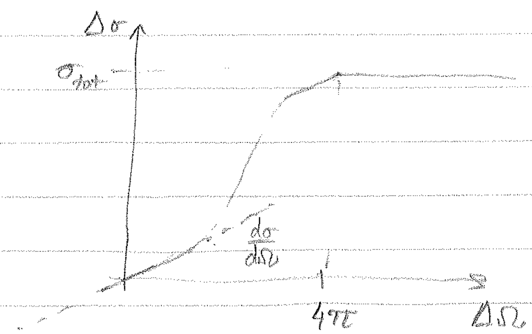
This is essentially a map of regions (on wall of incident particles) of size $\Delta\sigma$ onto regions of solid angle $\Delta\Omega$.

→ special case: fix position of $\Delta\sigma$, and adjust its size, then we can write this as:

$$\Delta\Omega = \Delta\Omega(\Delta\sigma) \leftarrow$$ and expect this to be a monotonically increasing function of $\Delta\sigma$.

$$\Rightarrow \quad \Delta\sigma = \Delta\sigma(\Delta\Omega)$$ "vary size of size of solid angle, and area on wall of incident particles changes".

For very small scattering solid angle $\Delta\Omega$, expect $\Delta\sigma$ to exhibit a linear dependence on $\Delta\Omega$: (Taylor expansion)



that is $\Delta\sigma(\Delta\Omega) = \frac{d\sigma}{d\Omega}\Delta\Omega + \mathcal{O}(\Delta\Omega^2)$

Plug into ⊛, so that

$$N(\Delta\Omega) = n_{inc.}\left(\frac{d\sigma}{d\Omega}\Delta\Omega + \mathcal{O}(\Delta\Omega^2)\right)$$

$$\Rightarrow \boxed{\frac{d\sigma}{d\Omega} = \frac{1}{n_{inc.}}\frac{N(\Delta\Omega)}{\Delta\Omega}}$$

differential cross section [calculated theoretically]

number of particles found in solid angle $\Delta\Omega$ normalized by $\Delta\Omega$ [determined experimentally]

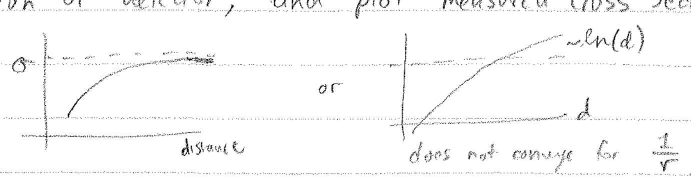
$$\sigma_{tot} = \frac{1}{n_{inc.}}\int \frac{N(\Delta\Omega)}{\Delta\Omega}\, d\Omega$$

Notes:

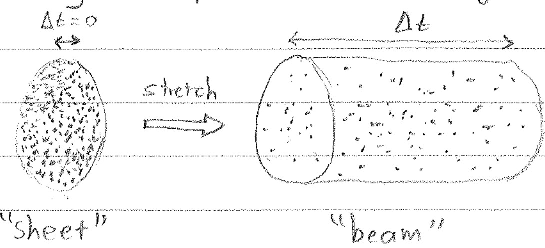
① Must make sure that the density of incident particles is uniform throughout the "infinite" wall (front)

② There might be practical difficulty with distinguishing an unscattered particle from a forward-scattered one.

$\Rightarrow$ solution: adjust position of detector, and plot measured cross section as function of distance.



For collider physics, make a small change:
Instead of running the experiment with a single sheet of incident particles,



run the experiment with an incident beam of particles,
so that we now speak of rates, per unit time.

$$\begin{pmatrix}\text{Number of scattered}\\ \text{particles per \underline{unit time}}\end{pmatrix} = \begin{pmatrix}\text{number of incident}\\ \text{particles per unit area}\\ \text{\underline{per unit time}}\end{pmatrix}\begin{pmatrix}\text{cross section}\end{pmatrix}$$

$$\frac{dN_{scatt}}{dt} = \frac{dn_{inc}}{dt} \times \sigma_{Tot} \,.$$

accelerator physicist: "Machine luminosity"
theoretical physicist: "Flux"

$$\frac{dN_{scatt}}{dt} = \mathcal{L}(t)\,\sigma_{Tot} \qquad \Rightarrow \qquad N_{scatt} = \left[\int dt\, \mathcal{L}(t)\right]\sigma_{Tot}$$

Total number of "events"

Then for the differential cross section,

$$\frac{dN_{scatt}}{d\Omega\, dt} = \mathcal{L}(t)\,\frac{d\sigma}{d\Omega}$$

Number of events registered by detector element at $d\Omega$ per unit time.

Time-dependent luminosity

Differential cross section.

For theorists, idealize scattering problem by considering a time independent luminosity (or flux):

$$\Rightarrow \quad \frac{d\sigma}{d\Omega} = \frac{1}{\text{Flux}}\left(\frac{dN_{scatt}}{d\Omega\, dt}\right)$$