Lippmann-Schwinger equation for $\hat{G}(\lambda)$.

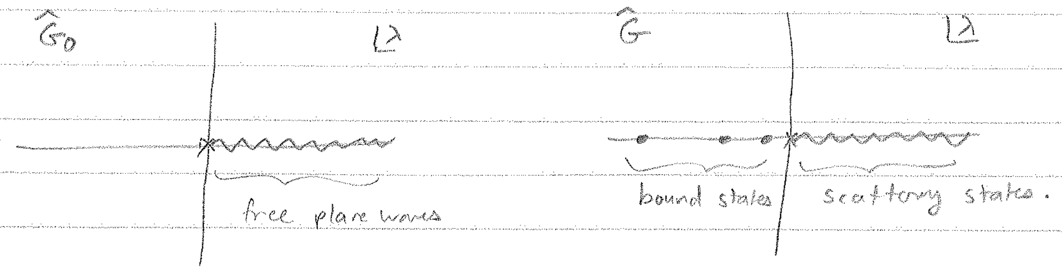
Hamiltonian: $H_0 = \dfrac{\vec{p}^{\,2}}{2m}$, $\quad H = \dfrac{\vec{p}^{\,2}}{2m} + V$

Introduce two Green's functions (Resolvent operators):

$$G_0(\lambda) = (\lambda - H_0)^{-1} \qquad \text{for free Hamiltonian}$$

$$G(\lambda) = (\lambda - H)^{-1} \qquad \text{for interacting Hamiltonian.}$$

Analytic structure of G_0 & G:



Solving for $\hat{G}(\lambda)$ is just as hard as solving eigenvalue problem.
However, we can relate $\hat{G}(\lambda)$ to the known $G_0(\lambda)$ and V.

Use identity:
$$\begin{aligned} A &= B + A - B \\ &= B + (BB^{-1})A - B(A^{-1}A) \\ &= B + B(B^{-1} - A^{-1})A \end{aligned}$$

Set $A = G(\lambda)$ and $B = G_0(\lambda)$
$A^{-1} = \lambda - H \qquad B^{-1} = \lambda - H_0$

$$\begin{aligned} G(\lambda) &= G_0(\lambda) + G_0(\lambda)(\cancel{\lambda} - H_0 - \cancel{\lambda} + H)\, G(\lambda) \\ &= G_0(\lambda) + G_0(\lambda)\underbrace{(H - H_0)}_{V} G(\lambda) \end{aligned}$$

$$\boxed{\hat{G}(\lambda) = \hat{G}_0(\lambda) + \hat{G}_0(\lambda)\,\hat{V}\,\hat{G}(\lambda)}$$

Lippmann-Schwinger equation for $\hat{G}(\lambda)$

also, $\boxed{\hat{G}(\lambda) = \hat{G}_0(\lambda) + \hat{G}(\lambda)\,\hat{V}\,\hat{G}_0(\lambda)}$

by interchanging $\hat{A}$ & $\hat{B}$.