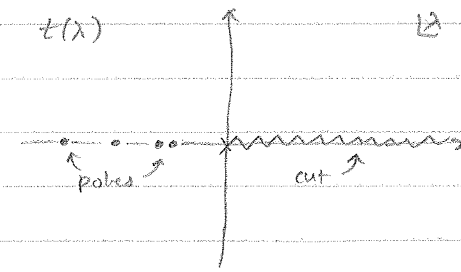
Transition Operator $\hat{t}(\lambda)$ [Off-shell $\hat{T}$ matrix]

Definition: $\hat{t}(\lambda) = \hat{V} + \hat{V}\hat{G}(\lambda)\hat{V} = \underbrace{\left(\hat{\mathbb{1}} + \hat{V}\hat{G}(\lambda)\right)\hat{V}}_{\text{[FORM I]}} = \underbrace{V\left(\hat{\mathbb{1}} + \hat{G}(\lambda)\hat{V}\right)}_{\text{[FORM II]}}$

Note: analytic structure of $\hat{t}(\lambda)$ is identical to that of $\hat{G}(\lambda)$



Multiply def^{n} FORM I at left by $\hat{G}_0(\lambda)$

$$\hat{G}_0(\lambda)\hat{t}(\lambda) = \underbrace{\left[\hat{G}_0 + \hat{G}_0\hat{V}\hat{G}(\lambda)\right]}_{G(\lambda)}\hat{V}$$

Lippmann-Schwinger for $\hat{G}(\lambda)$

$$\hat{G}_0(\lambda)\hat{t}(\lambda) = \hat{G}(\lambda)\hat{V} \qquad (*)$$

Multiply def^{n} FORM II at right by $\hat{G}_0(\lambda)$

$$\hat{t}(\lambda)\hat{G}_0(\lambda) = \hat{V}\underbrace{\left[\hat{G}_0(\lambda) + \hat{G}(\lambda)\hat{V}\hat{G}_0(\lambda)\right]}_{\hat{G}(\lambda)}$$

$$\hat{t}(\lambda)\hat{G}_0(\lambda) = \hat{V}\hat{G}(\lambda) \qquad (**)$$

$(*) \longleftrightarrow (**)$ equivalent

Applications

① Plug $(**)$ back into Lippmann-Schwinger eqn for $\hat{G}(\lambda)$:

$$\hat{G}(\lambda) = \hat{G}_0(\lambda) + \hat{G}_0(\lambda)\underbrace{\hat{V}G(\lambda)}_{\hat{t}(\lambda)\hat{G}_0(\lambda) \quad (**)}$$

$$\hat{G}(\lambda) = \hat{G}_0(\lambda) + \hat{G}_0(\lambda)\hat{t}(\lambda)\hat{G}_0(\lambda)$$

means: knowledge of $\hat{t}(\lambda)$ implies knowledge of $\hat{G}(\lambda)$.
(that is, the definition of $\hat{t}$ above leads to no loss of information about $\hat{G}$)

② Plug $(*)$ back into definition of $\hat{t}(\lambda)$:

$$\hat{t}(\lambda) = \hat{V} + \hat{V}\underbrace{\hat{G}(\lambda)\hat{V}}$$

$$\boxed{\hat{t}(\lambda) = \hat{V} + \hat{V}\,\hat{G}_0(\lambda)\,\hat{t}(\lambda)}$$

Lippmann-Schwinger equation for $\hat{t}(\lambda)$.