Yukawa Potential at Born level

- Concrete example illustrating behavior of Born-level amplitude.

$$V_{Yuk}(r) = \pm \frac{\hbar c \lambda^2}{4\pi} \frac{e^{-\mu r}}{r}$$

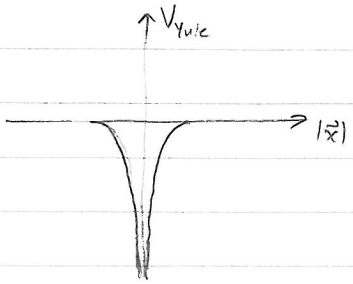
$\mu = \frac{m_\phi c}{\hbar}$

λ = dimensionless.

↖ coupling — reduced potential $U(r) = \frac{g e^{-\mu r}}{r}$

+ = repulsive
− = attractive.

$g = \frac{2m}{\hbar^2} \frac{\hbar c \lambda^2}{4\pi}$



$$f_{Born} = -\frac{2m}{\hbar^2} \int_0^\infty dr\, r^2 \frac{\sin(qr)}{qr} \left(-\frac{\hbar c \lambda^2}{4\pi} \frac{e^{-\mu r}}{r} \right) \quad \text{(attractive case)}$$

$$= \left(-\frac{2m}{\hbar^2} \right) \frac{-\hbar c \lambda^2}{4\pi} \frac{1}{q} \int_0^\infty dr \sin(qr)\, e^{-\mu r}$$

$$= + \frac{c}{\hbar} \frac{m}{2\pi} \frac{\lambda^2}{q} \left[\frac{q}{\vec{q}^2 + \mu^2} \right] = \frac{c}{\hbar} \frac{m}{2\pi} \frac{\lambda^2}{\vec{q}^2 + \mu^2}$$

$\vec{q}^2 = 4k^2 \sin^2(\theta/2)$

$$\left(\frac{d\sigma_{Yuk}}{d\Omega} \right)_{Born} = |f_{Born}|^2 = \frac{\left(\frac{c}{\hbar} \frac{m}{2\pi} \lambda^2 \right)^2}{(4k^2 \sin^2\theta/2 + \mu^2)}$$