Scattering on singular potentials

Consider form of potential:

$$V(r) = \frac{g}{r^m} C(r) \qquad C(0) = 1$$

$$m > 2 \quad \text{(singular)}$$

$$= r_0^{-2} \left(\frac{r_0}{r}\right)^m C(r)$$

g = coupling constant

r_0 = length scale of singular region. $\Rightarrow$ relationship: $\boxed{r_0^{m-2} = g}$

There is a major qualitative difference between attractive and repulsive potential (singular parts)

Repulsive potentials have direct physical interpretation

attractive ones have the feature of particles falling on center.

[In relativistic scattering, $V \sim \frac{1}{r}$ is already considered singular because diff. eq comes in $V^2 \sim \frac{1}{r^2}$].

Review ~

Schrödinger eq. becomes

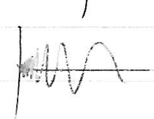
$$u''(r) - g r^{-m} u(r) = 0 \quad \text{in the vicinity of } r \approx 0.$$

Solutions:

physical solution is minus case.

$$u_\pm(r) = r e^{\pm\sqrt{g}/r} \quad \text{if } g > 0 \quad \text{(repulsive)}$$

$$u_\pm(r) = r \sin\left(|\sqrt{g}|/r \pm \pi/4\right) \quad \text{if } g < 0 \quad \text{(attractive)}$$

Both solutions: 

Infinite number of zeros $\Rightarrow \delta(r) \to \infty$

$\Rightarrow$ Infinite number of bound states.

$\longrightarrow$ From now, work only with repulsive potential since we can select the physical solution.