Representation theory of $SO(3)$ - Angular momentum

$(\eta = +1)$

Since the normalization is positive,

$$t(t+1) - m(m \pm 1) \geq 0$$

$$t(t+1) \geq m(m \pm 1)$$

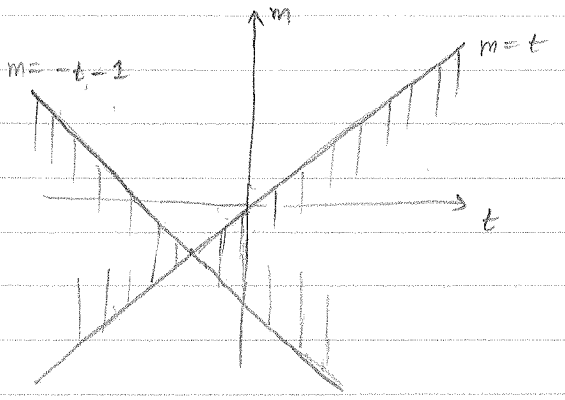
So, given t, what is the bound on m?

Rewrite inequality:

$$0 \geq m(m \pm 1) - t(t+1)$$
$$\geq m^2 \pm m - t^2 - t$$
$$\geq (m \mp t)(m \pm t \pm 1)$$ both cases (+ and −) need to be satisfied.

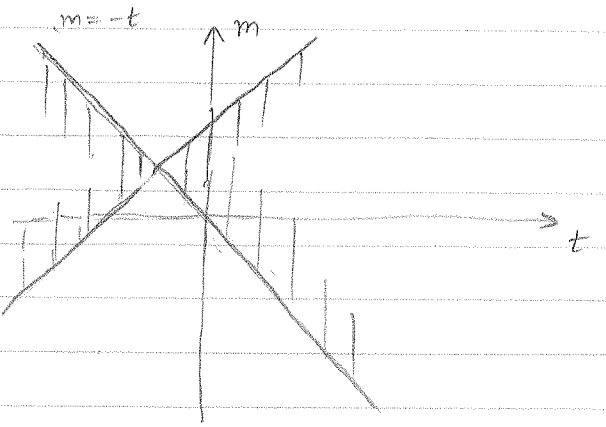
Case T_+

$$0 \geq (m-t)(m+t+1)$$



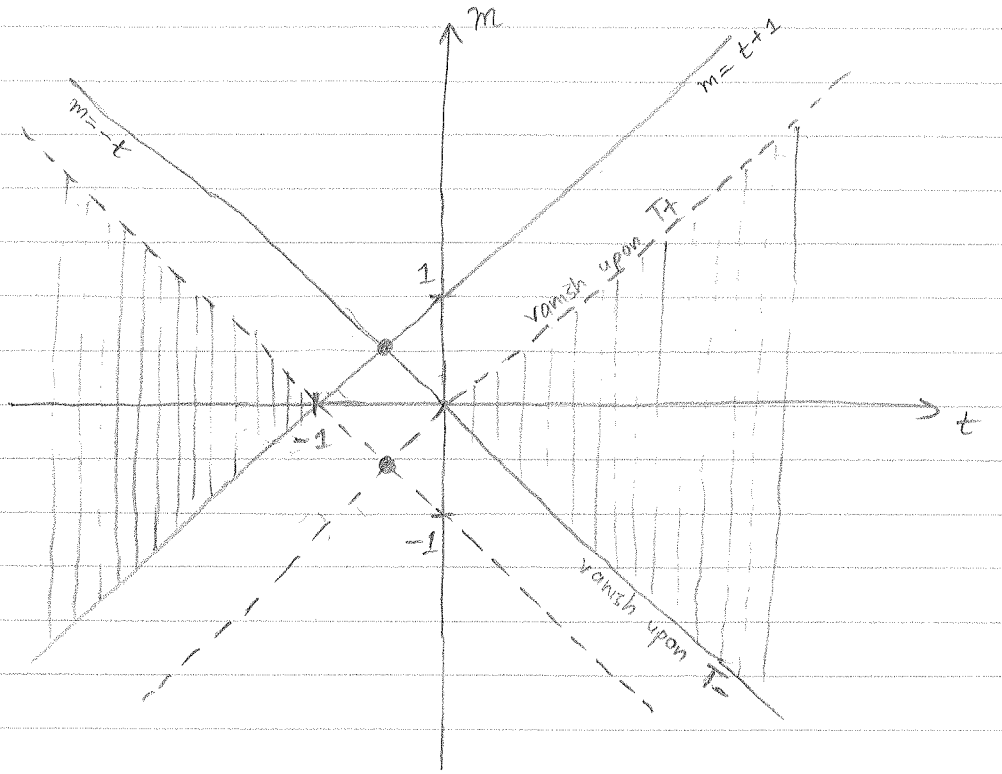
case T_-:

$$0 \geq (m+t)(m-t-1)$$



shaded region = values of m & t which satisfy equality.

Combine both conditions:



Lines (solid & dotted) correspond to zero normalization.

T_- T_+

In order for $\vec{T}^2$ eigenvalue $t(t+1)$ to be real, we must have either $t \in \mathbb{R}$ or $t = -1/2 + i\beta$ (β: real continuous parameter.)

The application of T_+ is to increase m by one unit.
The application of T_- is to decrease m by one unit.

$\Rightarrow$ in order for states $|t, m\rangle$ to stay within shaded region, t is required to be either INTEGERS (orbital series)
or HALF-ODD INTEGERS (spin series)