Scattering stationary states with spin:

Eigenvalue problem:

$$\hat{H}\psi = E\psi$$

where

$$\hat{H} = \hat{H}_0 + \hat{V}$$

no sum over ξ'

$$\sum_{\xi}\left[\frac{-\hbar^2}{2m}\nabla^2\delta_{\xi'\xi} + V(\vec{x})_{\xi'\xi}\right]\psi_{\vec{k},\xi} = E_{\vec{k},\xi'}\psi_{\vec{k},\xi'}$$

$$= E_{\vec{k},\xi}\delta_{\xi,\xi'}\psi_{\vec{k},\xi'}$$

$$(\hat{H}_0+\hat{V})\psi = E\psi$$

$$\sum_{\xi}\left[\frac{\hbar^2}{2m}\nabla^2 + E_{\vec{k},\xi}\right]\delta_{\xi,\xi'}\psi_{\vec{k},\xi'} = \sum_{\xi} V(\vec{x})_{\xi'\xi}\psi_{\vec{k},\xi}.$$

write it as

$$(-\hat{H}_0+E)\psi = +\hat{V}\psi$$

Define Green's function of $-\hat{H}_0+E$ as solution to S.E. with impulse:

$$(-H_0+E)G = \hat{\mathbb{1}}$$

Since $\hat{H}_0$ is diagonal in spin space, so is G.. Good.

Then all I need to do is write down the integral equation:

$$\psi_{\vec{k}}(\vec{x}) = \frac{1}{(2\pi)^{3/2}}e^{i\vec{k}\cdot\vec{x}} + \int d^3x'\, G_{\vec{k}}(\vec{x}-\vec{x}')\underset{\uparrow}{\left[V(\vec{x}')\right]}\psi_{\vec{k}'}(\vec{x}')$$

this is a potential matrix.

Therefore, Let me write $\psi_{\vec{k}}(\vec{x}')$ differently,

spin quantum numbers.

$$\boxed{\psi_{\vec{k}}(\vec{x},\xi') = \frac{1}{(2\pi)^{3/2}}e^{i\vec{k}\cdot\vec{x}}\chi_{\xi'} + \sum_{\xi}\int d^3x'\, G_{\vec{k}}(\vec{x}-\vec{x}')V(\vec{x}')_{\xi',\xi}\psi_{\vec{k}}(\vec{x}',\xi)}$$

stationary scattering state

direct subs. into S.E. yields equality. ✓

$$G_{\vec{k}}(\vec{x}-\vec{x}') = \frac{-m}{2\pi\hbar^2}\frac{e^{i|\vec{k}||\vec{x}-\vec{x}'|}}{|\vec{x}-\vec{x}'|}$$

$$\Rightarrow \psi_{\vec{k}}(\vec{x},\xi') \xrightarrow{r\to\infty} \frac{1}{(2\pi)^{3/2}}\left[e^{i\vec{k}\cdot\vec{x}}\chi_{\xi'} \underbrace{-\frac{m}{2\pi\hbar^2}\int d^3\vec{x}'\, e^{-i\vec{k}'\cdot\vec{x}'}\sum_{\xi}V(\vec{x})_{\xi',\xi}(2\pi)^{3/2}\psi_{\vec{k}}(\vec{x}',\xi)}_{f(\vec{k}'\xi'\leftarrow\vec{k}\,\xi)}\frac{e^{ikr}}{r}\right]$$