Parity transformation for one-particle states

Helicity Plane waves [NO INTRINSIC PARITY CONSIDERED]

Under reflection (mirror):

$$\hat{M}\,|\vec{k},\lambda\rangle = \hat{R}(\pi\hat{y})\,\hat{P}\,|\vec{k},\lambda\rangle$$

$$= \hat{R}(\pi\hat{y})\,|-\vec{k},-\lambda\rangle$$

$$= \hat{R}(\pi\hat{y})\,\hat{R}(\phi,\theta,-\phi)\,|-k_z;\, s, m_s=\lambda\rangle$$

$$= |\vec{k}\rangle \otimes \underbrace{\hat{R}_s(\pi\hat{y})\,\hat{R}_s(\phi,\theta,-\phi)\,|s;\, m_s=-\lambda\rangle}$$

$$= |\vec{k}\rangle \otimes (-1)^{-s+\lambda}\,|s,-\lambda\rangle$$

$$= (-1)^{-s+\lambda}\,|\vec{k},-\lambda\rangle$$

$\therefore$ Under parity:

$$\hat{P}\,|\vec{k},\lambda\rangle = \hat{R}^{\dagger}(\pi\hat{y})\,\hat{M}\,|\vec{k},\lambda\rangle$$

$$= (-1)^{-s+\lambda}\,\hat{R}^{\dagger}(\pi\hat{y})\,|\vec{k},-\lambda\rangle$$

Helicity spherical waves [NO INTRINSIC PARITY CONSIDERED]

$$|E,j,m_j,\lambda\rangle = \mathcal{N}\int d\Omega_{\vec{k}}\; D^{(j)*}_{m_j,\lambda}(\phi,\theta,-\phi)\,|\vec{k},\lambda\rangle$$

Parity:

$$\hat{P}\,|E,j,m_j,\lambda\rangle = \mathcal{N}\int d\Omega_{\vec{k}}\; D^{(j)*}_{m_j,\lambda}(\phi,\theta,-\phi)\,\hat{P}\,|\vec{k},\lambda\rangle$$

$\vdots$ Follow same steps

$$= (-1)^{-s+\cancel{\lambda}}(-1)^{j-\cancel{\lambda}}\,|E,j,m_j,-\lambda\rangle$$

$$= (-1)^{j-s}\,|E,j,m_j,-\lambda\rangle$$