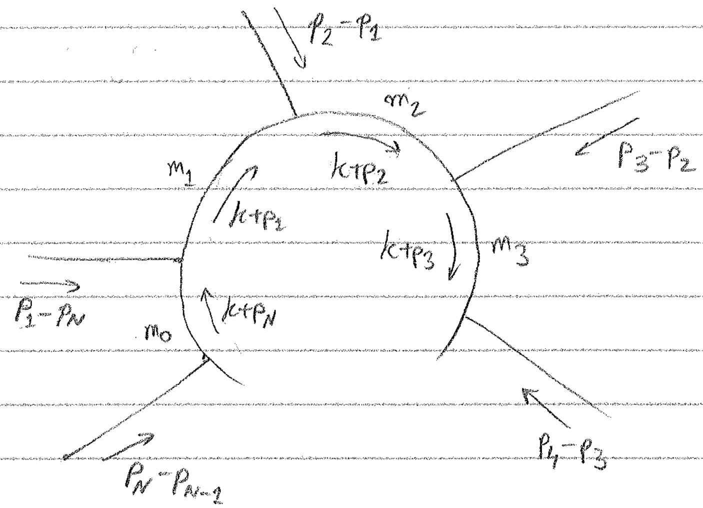
Definitions and conventions



All momenta incoming

Mom cons. $\Rightarrow$ $p_N = p_1 + p_2 + \dots + p_{N-1} = 0$.

int. masses: $\{m_0, m_1, m_2, \dots, m_{N-1}\}$.

Tensor integral:

$$T_N^{\mu_1 \cdots \mu_p} = \underbrace{\left(\frac{i e^{-\gamma_E \epsilon}}{(4\pi)^{d/2}}\right)^{-1}}_{N_\epsilon} \mu^{2\epsilon} \int \frac{d^dk}{(2\pi)^d} \frac{k^{\mu_1} k^{\mu_2} \cdots k^{\mu_p}}{[k^2 - m_0^2 + i\varepsilon][(k+p_1)^2 - m_1^2 + i\varepsilon] \cdots [(k+p_{N-1})^2 - m_{N-1}^2 + i\varepsilon]}$$

order of limits:

The $\epsilon \to 0$ limit must be performed before the $\varepsilon \to 0$ limit.

Convenient integration measures.

$$\mu^{2\epsilon}\frac{d^dk}{(2\pi)^d} = \left[\frac{i}{16\pi^2}\right](4\pi)^\epsilon\,\mu^{2\epsilon}\frac{d^dk}{i\pi^{d/2}}$$

$$= \underbrace{\frac{\Gamma(1-\epsilon)^2\Gamma(1+\epsilon)}{\Gamma(1-2\epsilon)}(4\pi)^\epsilon}_{r_\Gamma (4\pi)^\epsilon}\quad \underbrace{\mu^{2\epsilon}\frac{d^dk}{i\pi^{d/2}}\frac{\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)^2\Gamma(1+\epsilon)}}_{\text{Ellis' integration measure.}}$$

Alternatively (for Mathematicians)

$$\mu^{2\epsilon}\frac{d^dk}{(2\pi)^d} = \left[\frac{i}{16\pi^2}\right](4\pi)^\epsilon\,\mu^{2\epsilon}\frac{d^dk}{i\pi^{d/2}}$$

$$= \left[\frac{i}{16\pi^2}\right](4\pi)^\epsilon e^{-\gamma_E\epsilon}\quad\underbrace{e^{\gamma_E\epsilon}\mu^{2\epsilon}\frac{d^dk}{i\pi^{d/2}}}_{\text{Duhr's integration measure}}$$

[see lecture notes]

The difference.

$$\frac{\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)^2\Gamma(1+\epsilon)} \simeq 1 + \epsilon\gamma_E + \frac{\epsilon^2}{2}\left(\gamma_E^2 + \frac{\pi^2}{6}\right) + \ldots$$

$$e^{\gamma_E\epsilon} \simeq 1 + \epsilon\gamma_E + \frac{\epsilon^2}{2}\gamma_E^2$$

But $\frac{\pi^2}{6} = \zeta(2)$ is a period, and is admitted.

γ_E is NOT a period (over $\mathbb{Q}$)