Derivatives of tensor coefficient functions (unweighted, but also valid for weighted ones)
with respect to external invariants ↳see note below

A-functions

$$\frac{\partial}{\partial m_0} \underbrace{A_{0\cdots0}}_{2r} = \mu^{2\epsilon} \frac{(-1)^{1+r}}{2^r} e^{\gamma\epsilon} \Gamma(1-\tfrac{d}{2}-r) \frac{\partial}{\partial m_0} (m_0^2 - i\varepsilon)^{-1+\frac{d}{2}+r}$$

$$= \mu^{2\epsilon} \frac{(-1)^{1+r}}{2^r} e^{\gamma\epsilon} \underline{\Gamma(1-\tfrac{d}{2}-r)(-)(1-\tfrac{d}{2}-r)} (m_0^2 - i\varepsilon)^{-2+\frac{d}{2}+r} \; 2m_0$$

$$= \mu^{2\epsilon} \frac{(-1)^{0+r}}{2^r} e^{\gamma\epsilon} \Gamma(2-\tfrac{d}{2}-r) (m_0^2 - i\varepsilon)^{-2+\frac{d}{2}+r} \; 2m_0$$

Match against $\underbrace{A_{0\cdots0}}_{2(r-1)}$:

$$= \cancel{(4\pi\mu^2)^\epsilon} \frac{\cancel{(-1)^{0+r}}}{2^r} \cancel{\Gamma(2-\tfrac{d}{2}-r)} \, 2m_0 \left[\cancel{(4\pi\mu^2)^\epsilon} \frac{\cancel{(-1)^{0+r}}}{2^{r-1}} \cancel{\Gamma(2-\tfrac{d}{2}-r)} \right]^{-1} \underbrace{A_{0\cdots0}}_{2(r-1)}$$

$$= \frac{\cancel{2^{r-1}}}{\cancel{2^r}} \cancel{2} m_0 \underbrace{A_{0\cdots0}}_{2(r-1)}(m_0)$$

$$= m_0 \underbrace{A_{0\cdots0}}_{2(r-1)}(m_0) \quad \checkmark$$

To implement in Mathematica, use generalized Leibniz rule:

$$\frac{\partial}{\partial x^n}\left(f(x)\, g(x)\right) = \sum_{k=0}^{n} \binom{n}{k} f^{(k)}(x)\, g^{(n-k)}(x)$$

$$\therefore \frac{\partial^n}{\partial m_0^n} \underbrace{A_{0\cdots0}}_{2r}(m_0) = \frac{\partial^{n-1}}{\partial m_0^{n-1}} \left[m_0 \underbrace{A_{0\cdots0}}_{2(r-1)}(m_0) \right]$$

$$= \binom{n-1}{0} m_0 \frac{\partial^{n-1}}{\partial m_0^{n-1}} \underbrace{A_{0\cdots0}}_{2(r-1)} + \binom{n-1}{1} \frac{\partial^{n-2}}{\partial m_0^{n-2}} \underbrace{A_{0\cdots0}}_{2(r-2)}$$

NOTE: Valid for weighted Passarino-Veltman functions since weighted functions are linear combinations of unweighted ones with constant coefficients.

B functions

$$B_{\underbrace{0\cdots0}_{2r}\underbrace{1\cdots1}_{n}}(p^2, m_0, m_1) = \underbrace{\mu^{2\epsilon}e^{\gamma\epsilon}}_{\text{irrelevant}} \frac{(-1)^{2+r+n}}{2^r}\Gamma(2-\tfrac{d}{2}-r)\int_0^1 dx\ x^n\ \Delta^{-2+\frac{d}{2}+r}$$

$$\Delta = p^2x^2 + (-p^2+m_1^2-m_0^2)x + m_0^2 - i\epsilon$$

$$\frac{\partial}{\partial X} B_{\underbrace{0\cdots0}_{2r}\underbrace{1\cdots1}_{n}} = (-\text{const}-)(-)(2-\tfrac{d}{2}-r)\int_0^1 dx\ x^n\ \Delta^{-3+\frac{d}{2}+r}\ \frac{\partial}{\partial X}\Delta$$

$$= \underbrace{\mu^{2\epsilon}e^{\gamma\epsilon}}_{\text{irrelevant}} \frac{(-1)^{1+r+n}}{2^r}\Gamma(3-\tfrac{d}{2}-r)\int_0^1 dx\ x^n\left(\frac{\partial}{\partial X}\Delta\right)\Delta^{-3+\frac{d}{2}+r}$$

With respect to p^2:

$$\frac{\partial}{\partial p^2}\Delta = x^2 - x$$

$$\frac{\partial}{\partial p^2} B_{\underbrace{0\cdots0}_{2r}\underbrace{1\cdots1}_{n}} = (-\text{const}-)\left[\int_0^1 dx\ x^{2+n}\ \Delta^{-3+\frac{d}{2}+r} - \int_0^1 dx\ x^{n+1}\ \Delta^{-3+\frac{d}{2}+r}\right]$$

Match against $B_{\underbrace{0\cdots0}_{2(r-1)}\underbrace{1\cdots1}_{n+2}}$ & $B_{\underbrace{0\cdots0}_{2(r-1)}\underbrace{1\cdots1}_{n+1}}$

$$= \frac{(-1)^{1+r+n}}{2^r}\Gamma(3-\tfrac{d}{2}-r)\left[\frac{(-1)^{3+r+n}}{2^{r-1}}\Gamma(3-\tfrac{d}{2}-r)\right]^{-1} B_{\underbrace{0\cdots0}_{2(r-1)}\underbrace{1\cdots1}_{n+2}}$$

$$- \frac{(-1)^{1+r+n}}{2^r}\Gamma(3-\tfrac{d}{2}-r)\left[\frac{(-1)^{2+r+n}}{2^{r-1}}\Gamma(3-\tfrac{d}{2}-r)\right] B_{\underbrace{0\cdots0}_{2(r-1)}\underbrace{1\cdots1}_{n+1}}$$

$$= \frac{1}{2}\left(B_{\underbrace{0\cdots0}_{2(r-1)}\underbrace{1\cdots1}_{n+2}} + B_{\underbrace{0\cdots0}_{2(r-1)}\underbrace{1\cdots1}_{n+1}}\right)$$

Table of derivatives of Passarino-Veltman functions

A functions

$$\frac{\partial}{\partial m_0} A_{\underbrace{0\cdots0}_{2r}}(m_0) = m_0\, A_{\underbrace{0\cdots0}_{2(r-1)}}(m_0)$$

B functions

$$\frac{\partial}{\partial p^2} B_{\underbrace{0\cdots0}_{2r}\underbrace{1\cdots1}_{n}} = \frac{1}{2}\Big(B_{\underbrace{0\cdots0}_{2(r-1)}\underbrace{1\cdots1}_{n+2}} + B_{\underbrace{0\cdots0}_{2(r-1)}\underbrace{1\cdots1}_{n+1}} \Big)$$

$$\frac{\partial}{\partial m_0} B_{0\cdots0\,1\cdots1} = m_0 \Big(B_{\underbrace{0\cdots0}_{2(r-1)}\underbrace{1\cdots1}_{n+1}} + B_{\underbrace{0\cdots0}_{2(r-1)}\underbrace{1\cdots1}_{n}} \Big)$$

$$\frac{\partial}{\partial m_1} B_{0\cdots0\,1\cdots1} = -m_1\, B_{\underbrace{0\cdots0}_{2(r-1)}\underbrace{1\cdots1}_{n+1}}$$

C functions $(p_1^2, q^2, p_2^2; m_0, m_1, m_2)$

$$\frac{\partial}{\partial p_1^2} C_{\underbrace{0\cdots0}_{2r}\underbrace{1\cdots1}_{n_1}\underbrace{2\cdots2}_{n_2}} = \frac{1}{2}\Big(C_{\underbrace{0\cdots0}_{2(r-1)}\underbrace{1\cdots1}_{n_1+2}\underbrace{2\cdots2}_{n_2}} + C_{\underbrace{0\cdots0}_{2(r-1)}\underbrace{1\cdots1}_{n_1+1}\underbrace{2\cdots2}_{n_2+1}} + C_{\underbrace{0\cdots0}_{2(r-1)}\underbrace{1\cdots1}_{n_1+1}\underbrace{2\cdots2}_{n_2}} \Big)$$

$$\frac{\partial}{\partial q^2} C_{0\cdots0\,1\cdots1\,2\cdots2} = -\frac{1}{2}\, C_{\underbrace{0\cdots0}_{2(r-1)}\underbrace{1\cdots1}_{n_1+1}\underbrace{2\cdots2}_{n_2+1}}$$

$$\frac{\partial}{\partial p_2^2} C_{0\cdots0\,1\cdots1\,2\cdots2} = \frac{1}{2}\Big(C_{\underbrace{0\cdots0}_{2(r-1)}\underbrace{1\cdots1}_{n_1}\underbrace{2\cdots2}_{n_2+2}} + C_{\underbrace{0\cdots0}_{2(r-1)}\underbrace{1\cdots1}_{n_1+1}\underbrace{2\cdots2}_{n_2+1}} + C_{\underbrace{0\cdots0}_{2(r-1)}\underbrace{1\cdots1}_{n_1}\underbrace{2\cdots2}_{n_2+1}} \Big)$$

$$\frac{\partial}{\partial m_0} C_{0\cdots0\,1\cdots1\,2\cdots2} = m_0 \Big(C_{\underbrace{0\cdots0}_{2(r-1)}\underbrace{1\cdots1}_{n_1+1}\underbrace{2\cdots2}_{n_2}} + C_{\underbrace{0\cdots0}_{2(r-1)}\underbrace{1\cdots1}_{n_1}\underbrace{2\cdots2}_{n_2+1}} + C_{\underbrace{0\cdots0}_{2(r-1)}\underbrace{1\cdots1}_{n_1}\underbrace{2\cdots2}_{n_2}} \Big)$$

$$\frac{\partial}{\partial m_1} C_{0\cdots0\,1\cdots1\,2\cdots2} = -m_1\, C_{\underbrace{0\cdots0}_{2(r-1)}\underbrace{1\cdots1}_{n_1+1}\underbrace{2\cdots2}_{n_2}}$$

$$\frac{\partial}{\partial m_2} C_{0\cdots0\,1\cdots1\,2\cdots2} = -m_2\, C_{\underbrace{0\cdots0}_{2(r-1)}\underbrace{1\cdots1}_{n_1}\underbrace{2\cdots2}_{n_2+1}}$$