

Behavior of Passarino-Veltman functions at infinity

It is always possible to convert the problem of analyzing a Passarino-Veltman function at infinity into one of analyzing it near zero. Consider the behavior of  $B_{0 \dots 0 1 \dots 1}(s; m_0, m_1)$  near  $m_0 \rightarrow \infty$ :

$$B_{\underbrace{0 \dots 0}_{2r} \underbrace{1 \dots 1}_n}(s; m_0, m_1) = [\text{Prefactor}] \mu^{2\epsilon} \int_0^1 dx \frac{x^n}{[s x^2 + (-s + m_1^2 - m_0^2)x + m_0^2 - i\epsilon]} e^{-\Gamma}$$

Factor out  $m_0$  from denominator:

$$\begin{aligned} B_{\underbrace{0 \dots 0}_{2r} \underbrace{1 \dots 1}_n}(s; m_0, m_1) &= [\text{prefactor}] \mu^{2\epsilon} \frac{1}{(m_0^2)^\epsilon} \int_0^1 dx \frac{x^n}{\left[ \frac{s}{m_0^2} x^2 + \left( \frac{-s + m_1^2 - m_0^2}{m_0^2} \right) x + 1 - \frac{i\epsilon}{m_0^2} \right] e^{-\Gamma}} \\ &= \frac{(m_0^2)^\epsilon}{(m_0^2)^\epsilon} B_{\underbrace{0 \dots 0}_{2r} \underbrace{1 \dots 1}_n} \left( \frac{s}{m_0^2}, 1, \frac{m_1}{m_0} \right). \end{aligned}$$

All that is needed is the behavior of  $B_{0 \dots 0 1 \dots 1}$  near  $(0, 1, 0)$ .  
since  $m_0 \rightarrow \infty$  means  $\frac{s}{m_0^2}, \frac{m_1}{m_0} \rightarrow 0$ .

More generally, to determine the  $x^2 \rightarrow \infty$  behavior of a Passarino-Veltman function, just rescale the rest of its arguments:

$$\int \frac{\{v_0 \dots v_{N-1}\} d^d}{\underbrace{0 \dots 0}_{2r} \underbrace{1 \dots 1}_{n_1} \dots \underbrace{1 \dots 1}_{n_{N-1}}}(s, \dots, x, \dots, m, \dots) = x^{\Gamma + \frac{d}{2} - v_0 - \dots - v_{N-1}} \int \frac{\{v_0 \dots v_{N-1}\} d^d}{\underbrace{0 \dots 0}_{2r} \underbrace{1 \dots 1}_{n_1} \dots \underbrace{1 \dots 1}_{n_{N-1}}}\left(\frac{s}{x^2}, \dots, 1, \dots, \frac{m}{x}\right)$$

Note: The mass dimension of a Passarino-Veltman function is  $[\Gamma] = \Gamma + \frac{d}{2} - v_0 - \dots - v_{N-1}$ .