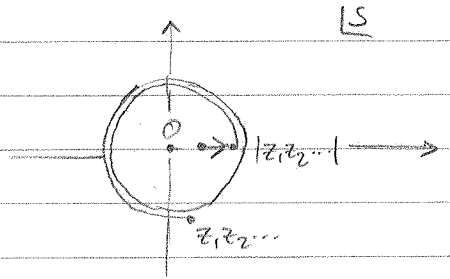


Continued natural logarithm

Definition:

$$\text{Ln}(z_1, z_2, \dots) = \int_1^{z_1 z_2 \dots} \frac{ds}{s}$$



for $-\pi < \arg(z_1), \arg(z_2), \dots \leq \pi$

i.e. complex numbers z_1, z_2, \dots are allowed to vary only on first sheet.

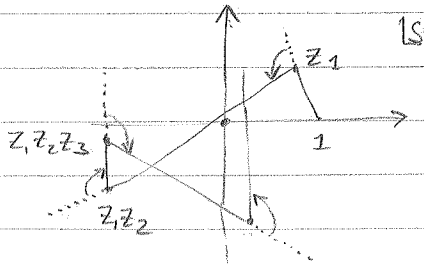
Contour of integration:

- starts at $s=+1$, goes to $s = |z_1 z_2 \dots|$, wraps around pole clockwise (if $\arg(z_1) + \arg(z_2) + \dots > 0$) or counterclockwise (if $\arg(z_1) + \arg(z_2) + \dots < 0$) ending at $s = z_1 z_2 \dots$

Numerical evaluation:

① Deform contour of integration to a series of segments:

$$1 \rightarrow z_1, \quad z_1 \rightarrow z_1 z_2, \quad z_1 z_2 \rightarrow z_1 z_2 z_3, \quad \dots$$



$$\text{Ln}(z_1, z_2, \dots) = \int_1^{z_1} \frac{ds}{s} + \int_{z_1}^{z_1 z_2} \frac{ds}{s} + \int_{z_1 z_2}^{z_1 z_2 z_3} \frac{ds}{s} + \dots$$

$$= \ln(z_1) + \ln\left(\frac{z_1 z_2}{z_1}\right) + \ln\left(\frac{z_1 z_2 z_3}{z_1 z_2}\right) + \dots$$

$$= \ln(z_1) + \ln(z_2) + \ln(z_3) + \dots$$

