Scalar A_0 function in $d = 4 - 2\epsilon$ dimensions

Integral to d_0:

$$\mu^{2\epsilon} \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2 - m_0^2 + i\epsilon}$$

$$= \mu^{2\epsilon} \frac{(-1)^1 i}{(4\pi)^{d/2}} \frac{1}{\Gamma(1)} \Gamma\left(1 - \frac{d}{2}\right) \left(\frac{1}{m_0^2}\right)^{1 - d/2}$$

$$= \frac{i}{(4\pi)^2} \left[m_0^2 \left(\frac{1}{\epsilon} - \gamma_E + \ln 4\pi \right) + m_0^2 \left(-\ln\left(\frac{m_0^2}{\mu^2}\right) + 1 \right) \right] + O(\epsilon)$$

$$= \frac{i}{(4\pi)^2} A_0(m_0)$$

No imaginary part

No IR divergence.