Non-recursive (iterative) reduction for $B_{\underbrace{0\cdots0}_{2r}\underbrace{1\cdots1}_{n}}$, $r\neq 0$ in $d=4-2\epsilon$ dimensions.

Definition:

$$B_{\underbrace{0\cdots0}_{2r}\underbrace{1\cdots1}_{n}}(p^2,m_0,m_1) = \mu^{2\epsilon}\,\frac{(-1)^{2+r+n}}{2^r}\,e^{\gamma\epsilon}\,\Gamma(\epsilon-r)\int_0^1 dx\,\frac{x^n}{(\Delta)^{2-\frac{d}{2}-r}}$$

where $\Delta = p^2x^2 + (-p^2+m_1^2-m_0^2)x + m_0^2 - i\varepsilon$.

If $r>0$, then bring Δ into numerator:

$$B_{0\cdots01\cdots1} = \mu^{2\epsilon}\,\frac{(-1)^{2+r+n}}{2^r}\,e^{\gamma\epsilon}\,\Gamma(\epsilon-r)\int_0^1 dx\,\frac{x^n\Delta^r}{\Delta^{2-d/2}}$$

Perform trinomial expansion:

$$\Delta^r = \sum_{j,k}\binom{r}{j,\,k,\,r-j-k}(p^2)^j x^{2j}\,(-p^2+m_1^2-m_0^2)^k x^k\,(m_0^2)^{r-j-k}$$

$$B_{0\cdots01\cdots1} = \cancel{\mu^{2\epsilon}}\,\frac{(-1)^{\cancel{2}+r+\cancel{n}}}{2^r}\,\cancel{e^{\gamma\epsilon}}\,\Gamma(\epsilon-r)$$

$$\times\sum_{j,k}\binom{r}{j,\,k,\,r-j-k}(p^2)^j(-p^2+m_1^2-m_0^2)^k(m_0^2)^{r-j-k}\int_0^1 dx\,\frac{x^{n+2j+k}}{\Delta^{2-d/2}}$$

Identify integral as B function with $r'=0$ and $n'=n+2j+k$

$$\int_0^1 dx\,\frac{x^{n+2j+k}}{\Delta^{2-d/2}} = \left[\cancel{\mu^{2\epsilon}}\,\frac{(-1)^{2+n+2j+k}}{2^0}\,\cancel{e^{\gamma\epsilon}}\,\Gamma(\epsilon)\right]^{-1} B_{\underbrace{1\cdots1}_{n+2j+k}}$$

$$\boxed{B_{\underbrace{0\cdots0}_{2r}\underbrace{1\cdots1}_{n}} = \frac{\Gamma(\epsilon-r)}{2^r\,\Gamma(\epsilon)}\sum_{j,k}^{r}(-1)^{r+k}\binom{r}{j,\,k,\,r-j-k}(p^2)^j(-p^2+m_1^2-m_0^2)^k(m_0^2)^{r-j-k}\,B_{\underbrace{1\cdots1}_{n+2j+k}}}$$

Gives an iterative (non-recursive) expression for $B_{\underbrace{0\cdots0}_{2r}\underbrace{1\cdots1}_{n}}$ $(r>0)$ in terms of $B_{\underbrace{1\cdots1}_{n}}$

Obtain $B_{\underbrace{1\cdots1}_{n}}$ by direct integration.