Hiren H. Patel
hnpatel.net

Lorentz decomposition of tensor C-functions

Def$^{\underline{n}}$

$$C^{\mu_1\cdots\mu_P}(p_1, p_2; m_0, m_1, m_2) = \mathcal{N}_\epsilon\, \mu^{2\epsilon}\int\frac{d^dk}{(2\pi)^d}\frac{k^{\mu_1}\cdots k^{\mu_P}}{(k^2-m_0^2)((k+p_1)^2-m_1^2)((k+p_2)^2-m_2^2)}$$

Lorentz covariance permits a tensor decomposition in terms of

- external momenta &
- metric tensor

$$C^\mu = p_1^\mu C_1 + p_2^\mu C_2$$

$$C^{\mu\nu} = p_1^\mu p_1^\nu C_{11} + p_1^\mu p_2^\nu C_{12} + p_2^\mu p_1^\nu C_{21} + p_2^\mu p_2^\nu C_{22} + g^{\mu\nu} C_{00}$$

PROPERTY 1

Tensor function $C^{\mu_1\mu_2\cdots}$ is a totally symmetric tensor.

$\Rightarrow$ coefficients are also symmetric:

$C_{12} = C_{21}$ (and $C_{121} = C_{211} = C_{112}$, or $C_{100} = C_{010} = C_{001}$ ...)

$$\therefore\; C^{\mu\nu} = p_1^\mu p_1^\mu C_{11} + (p_1^\mu p_2^\nu + p_1^\nu p_2^\mu) C_{12} + p_2^\mu p_2^\nu C_{22} + g^{\mu\nu} C_{00}$$

Each of these is a totally symmetric tensor, expressed most compactly:

$$= \{[p_1]^2\}^{\mu\nu} C_{11} + \{[p_1][p_2]\}^{\mu\nu} C_{12} + \{[p_2]^2\}^{\mu\nu} C_{22} + \{[g]\}^{\mu\nu} C_{00}$$

So, all that is needed to specify a tensor coefficient function is the multiplicity number of each unique momentum & metric tensor:

$C_{\underbrace{0\cdots0}_{2r}\underbrace{1\cdots1}_{n_1}\underbrace{2\cdots2}_{n_2}}$ is the function multiplying $\{[p_1]^{n_1}[p_2]^{n_2}[g]^r\}^{\mu_1\cdots\mu_P}$

(and $n_1+n_2+2r=P$ for this to make sense).

Further invariance: PROPERTY 2

There is nothing unique about the labeling of momenta.

Can exchange multiplicity numbers n_1 & n_2, provided the 4-momenta p_1^μ, p_2^μ and masses m_1, m_2 are exchanged)

(meaning $p_1^2 \leftrightarrow p_2^2$; q^2 fixed; $m_1^2 \leftrightarrow m_2^2$).

That is: $C_{\underbrace{0\cdots0}_{2r}\underbrace{1\cdots1}_{n_1}\underbrace{2\cdots2}_{n_2}}(p_1^2, q^2, p_2^2; m_0, m_1, m_2) = C_{\underbrace{0\cdots0}_{2r}\underbrace{1\cdots1}_{n_2}\underbrace{2\cdots2}_{n_1}}(p_2^2, q^2, p_1^2; m_0, m_2, m_1)$.