Reduction case IV $\quad z_{ij}=0 \quad$ (but not f_i)

If all elements of $p_i \cdot p_j$ is vanishing,

Primary reduction formula ① becomes:

$$n_k D_{\hat{k}00\underbrace{0\cdots0}_{2r}\underbrace{1\cdots1}_{n_1}\underbrace{2\cdots2}_{n_2}\underbrace{3\cdots3}_{n_3}} = \frac{1}{2}\Big[\delta_{n_k 0}\, C_{\underbrace{0\cdots0}_{2r}\underbrace{1\cdots1}_{n_{1(k)}}\underbrace{2\cdots2}_{n_{2(k)}}}(\hat{D}_k) - C_{\underbrace{0\cdots0}_{2r}\underbrace{1\cdots1}_{n_1}\underbrace{2\cdots2}_{n_2}\underbrace{3\cdots3}_{n_3}}(\hat{D}_0) - f_k D_{\underbrace{0\cdots0}_{2r}\underbrace{1\cdots1}_{n_1}\underbrace{2\cdots2}_{n_2}\underbrace{3\cdots3}_{n_3}}\Big]$$

solve for this.

$$f_k D_{0\cdots0 1\cdots1 2\cdots2 3\cdots3} = -2n_k D_{\hat{k}000\cdots0 1\cdots1 2\cdots2 3\cdots3} + \delta_{n_k 0}\, C_{0\cdots0 1\cdots1 2\cdots2}(\hat{D}_k) - C_{0\cdots0 1\cdots1 2\cdots2 3\cdots3}(\hat{D}_0)$$

To implement in Mathematica, put $r=0$:

$$D_{\underbrace{1\cdots1}_{n_1}\underbrace{2\cdots2}_{n_2}\underbrace{3\cdots3}_{n_3}} = \frac{1}{f_k}\Big[-2n_k D_{\hat{k}00\underbrace{1\cdots1}_{n_1}\underbrace{2\cdots2}_{n_2}\underbrace{3\cdots3}_{n_3}} + \delta_{n_k 0}\, C_{\underbrace{1\cdots1}_{n_{1(k)}}\underbrace{2\cdots2}_{n_{2(k)}}}(\hat{D}_k) - C_{\underbrace{1\cdots1}_{n_1}\underbrace{2\cdots2}_{n_2}\underbrace{3\cdots3}_{n_3}}(\hat{D}_0)\Big] \qquad [1]$$

choose simplest non-zero f_k

Primary reduction formula ② becomes:

$$(d+2(r+n_1+n_2+n_3))\, D_{000\cdots0 1\cdots1 2\cdots2 3\cdots3} = C_{0\cdots0 1\cdots1 2\cdots2 3\cdots3}(\hat{D}_0) + m_0^2 D_{0\cdots0 1\cdots1 2\cdots2 3\cdots3} \qquad (**)$$

To implement in Mathematica, put $r \to r-1$

$$(d+2(r-1+n_1+n_2+n_3))\, D_{0\cdots0 1\cdots1 2\cdots2 3\cdots3} = C_{\underbrace{0\cdots0}_{2(r-1)} 1\cdots1 2\cdots2 3\cdots3}(\hat{D}_0) + m_0^2 D_{\underbrace{0\cdots0}_{2(r-1)} 1\cdots1 2\cdots2 3\cdots3}$$

$(2-2\epsilon+2r+2n_1+2n_2+2n_3)\, D$

$$\therefore\ D_{\underbrace{0\cdots0}_{2r}\underbrace{1\cdots1}_{n_1}\underbrace{2\cdots2}_{n_2}\underbrace{3\cdots3}_{n_3}} = \frac{1}{2(1+r+n_1+n_2+n_3)}\Big[C_{\underbrace{0\cdots0}_{2(r-1)}\underbrace{1\cdots1}_{n_1}\underbrace{2\cdots2}_{n_2}\underbrace{3\cdots3}_{n_3}}(\hat{D}_0) + m_0^2 D_{\underbrace{0\cdots0}_{2(r-1)}\underbrace{1\cdots1}_{n_1}\underbrace{2\cdots2}_{n_2}\underbrace{3\cdots3}_{n_3}}\Big]$$

$$+ \frac{1}{1+r+n_1+n_2+n_3} D_{\underbrace{0\cdots0}_{2r}\underbrace{1\cdots1}_{n_1}\underbrace{2\cdots2}_{n_2}\underbrace{3\cdots3}_{n_3}}\Big|_{UV\ Div}, \quad r \geq 1 \qquad [2]$$