

Reduction formulae for negative rank $r < 0$

If $\det Z \neq 0$ and $\det X \neq 0$, (If $\det Z = 0$, use DD reduction)

Start Passarino-Veltman reduction formula in matrix form:

$$\underbrace{\begin{pmatrix} 2m_0^2 & f_1 & f_2 & f_3 \\ f_1 & 2p_1 p_1 & 2p_1 p_2 & 2p_1 p_3 \\ f_2 & & 2p_2 p_2 & 2p_2 p_3 \\ f_3 & & & 2p_3 p_3 \end{pmatrix}}_X \begin{pmatrix} D_{0 \dots 0 1 \dots 1 2 \dots 2 3 \dots 3} \\ D_{0 \dots 0 1 \dots 1 2 \dots 2 3 \dots 3} \\ D_{0 \dots 0 1 \dots 1 2 \dots 2 3 \dots 3} \\ D_{0 \dots 0 1 \dots 1 2 \dots 2 3 \dots 3} \end{pmatrix} = \begin{pmatrix} 2(d-3+2r+n_1+n_2+n_3) D_{0 \dots 0 1 \dots 1 2 \dots 2 3 \dots 3} - C_{0 \dots 0 1 \dots 1 2 \dots 2 3 \dots 3} \\ 2(r+1) \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

Multiply at left by $X^{-1} = \frac{1}{\det X} \tilde{X}$ and take first equation:

$$\underbrace{D_{0 \dots 0 1 \dots 1 2 \dots 2 3 \dots 3}}_{2r \quad n_1 \quad n_2 \quad n_3} = \frac{1}{\det X} \left[\tilde{X}_{00} \left(2 \overset{1-2e}{(d-3+2r+n_1+n_2+n_3)} \underbrace{D_{0 \dots 0 1 \dots 1 2 \dots 2 3 \dots 3}}_{2(r+1)} - \underbrace{C_{0 \dots 0 1 \dots 1 2 \dots 2 3 \dots 3}}_{2r \quad n_1 \quad n_2 \quad n_3} (\hat{D}_0) \right) + \sum_{j=1}^3 \tilde{X}_{0j} \left(\delta_{n_j 0} \underbrace{C_{0 \dots 0 1 \dots 1 2 \dots 2}}_{n_j(j) \quad n_j(j)} (\hat{D}_j) - \underbrace{C_{0 \dots 0 1 \dots 1 2 \dots 2 3 \dots 3}}_{2(r+1)} (\hat{D}_0) - 2n_j \underbrace{D_{0 \dots 0 1 \dots 1 2 \dots 2 3 \dots 3}}_{2(r+1)} \right) \right]$$

$$\tilde{X}_{00} = 8 \det Z \quad \checkmark$$

$$\tilde{X}_{0j} = - \sum_{n=1}^3 4 \tilde{z}_{jn} f_n \quad \checkmark$$

If $\det X = 0$, Landau condition for leading singularity is satisfied.