Feynman-parametric representation of one-loop tensor integrals with repeated propagators

Definition:

$$T_N^{\mu_1\cdots\mu_p}(d;\{\nu_1,\dots,\nu_N\}) = \left(\frac{ie^{-\gamma_E\epsilon}}{(4\pi)^{d/2}}\right)^{-1}\mu^{2\epsilon}\int\frac{d^dk}{(2\pi)^d}\,\frac{k^{\mu_1}\cdots k^{\mu_p}}{D_1^{\nu_1}(k)\cdots D_N^{\nu_N}(k)}$$

Combine denominators (Feynman's trick)

$$= \left(\frac{ie^{-\gamma_E\epsilon}}{(4\pi)^{d/2}}\right)^{-1}\int_0^1 dx_1\cdots dx_N\;\delta\Big(1-\sum_{i=1}^N x_i\Big)\frac{\Gamma(\nu_1+\dots+\nu_N)}{\Gamma(\nu_1)\cdots\Gamma(\nu_N)}\,x_1^{\nu_1-1}\cdots x_N^{\nu_N-1}$$

$$\times\;\mu^{2\epsilon}\int\frac{d^dk}{(2\pi)^d}\,\frac{k^{\mu_1}\cdots k^{\mu_p}}{[x_1D_1+\dots+x_ND_N]^{\nu_1+\dots+\nu_N}}$$

Evaluate d^dk integral

$$= \left(\frac{ie^{-\gamma_E\epsilon}}{(4\pi)^{d/2}}\right)^{-1}\int_0^1 dx_1\cdots dx_N\;\delta\Big(1-\sum_{i=1}^N x_i\Big)\frac{\Gamma(\nu_1+\dots+\nu_N)}{\Gamma(\nu_1)\cdots\Gamma(\nu_N)}\,x_1^{\nu_1-1}\cdots x_N^{\nu_N-1}$$

$$\times\;\frac{\mu^{2\epsilon}\,i}{(4\pi)^{d/2}}\,\frac{1}{\Gamma(\nu_1+\dots+\nu_N)}\sum_{r=0}^{p/2}\;\sum_{\substack{\{n_1,\dots,n_N\}=0\\ n_1+\dots+n_N+2r=p}}^{p-2r}\Bigg[\frac{(-1)^{\nu_1+\dots+\nu_N+p-r}}{2^r}\,\Gamma\Big(\nu_1+\dots+\nu_N-\frac{d}{2}-r\Big)$$

$$\times\;x_1^{n_1}\cdots x_N^{n_N}\Big\{[p_1]^{n_1}\cdots[p_N]^{n_N}[g]^r\Big\}^{\mu_1\cdots\mu_p}\left(\frac{1}{\Delta}\right)^{\nu_1+\dots+\nu_N-r-\frac{d}{2}}\Bigg]$$

$$\boxed{\begin{aligned}T_N^{\mu_1\cdots\mu_p}(d;\{\nu_1,\dots,\nu_N\}) &= \sum_{r=0}^{p/2}\;\sum_{\substack{\{n_1,\dots,n_N\}=0\\ n_1+\dots+n_N+2r=p}}^{p-2r}\Bigg[\Big\{[p_1]^{n_1}\cdots[p_N]^{n_N}[g]^r\Big\}^{\mu_1\cdots\mu_p}\\ &\times\mu^{2\epsilon}\,\frac{(-1)^{\nu_1+\dots+\nu_N+p-r}}{2^r}\,\frac{e^{\gamma_E\epsilon}\,\Gamma(\nu_1+\dots+\nu_N-\frac{d}{2}-r)}{\Gamma(\nu_1)\cdots\Gamma(\nu_N)}\int_0^1 dx_1\cdots dx_N\;x_1^{\nu_1+n_1-1}\cdots x_N^{\nu_N+n_N-1}\,\delta\Big(1-\sum_i x_i\Big)\\ &\times\left(\frac{1}{\Delta}\right)^{\nu_1+\dots+\nu_N-r-\frac{d}{2}}\Bigg]\end{aligned}}$$

Tensor coefficient function

$T_N(d;\{\nu_1,\dots,\nu_N\})_{\underbrace{0\cdots0}_{2r}\,\underbrace{1\cdots1}_{n_1}\,\cdots\,\underbrace{N\cdots N}_{n_N}}$ is quantity in parenthesis (including $(4\pi\mu^2)^\epsilon$).