Connection to standard coefficient functions of unit weight in $d = 4-2\epsilon$ dimensions

(take $p_N = 0 \Rightarrow n_N = 0$) put $\nu_N \equiv \nu_0$

$$T^{\{\nu_0 \cdots \nu_{N-1}\};\, d}_{\underbrace{0\cdots 0}_{2r}\underbrace{1\cdots 1}_{n_1}\cdots\underbrace{(N-1)\cdots(N-1)}_{n_{N-1}}} = \mu^{2\epsilon}\, \underline{(-1)^{\nu_0+\ldots+\nu_{N-1}+n_1+\ldots+n_{N-1}+r}}\; \frac{e^{\gamma_E \epsilon}\,\Gamma(\nu_0+\ldots+\nu_{N-1}-\frac{d}{2}-r)}{\Gamma(\nu_0)\cdots\Gamma(\nu_{N-1})}$$

$$\times \int_0^1 dx_1 \cdots dx_{N-1}\; x_1^{\nu_1+n_1-1} \cdots x_{N-1}^{\nu_{N-1}+n_{N-1}-1} \underbrace{(1-x_1-\ldots-x_{N-1})^{\nu_0-1}}_{\text{multinomial expansion}} \left(\frac{1}{\Delta}\right)^{-r-\frac{d}{2}+\nu_0+\ldots+\nu_{N-1}}$$

Multinomial expansion:

$$(1-x_1-\ldots-x_{N-1})^{\nu_0-1} = \sum_{j_1\cdots j_{N-1}}^{\nu_0-1} \binom{\nu_0-1}{j_1,\ldots,j_{N-1},\,\nu_0-1-j_1-\ldots-j_{N-1}} (-x_1)^{j_1}\cdots(-x_{N-1})^{j_{N-1}}$$

$$= \underline{\sum_{j_1\cdots j_{N-1}}^{\nu_0-1} \binom{\nu_0-1}{j_1,\ldots,j_{N-1},\,\nu_0-1-j_1-\ldots-j_{N-1}} (-1)^{j_1+\ldots+j_{N-1}}\; x_1^{j_1}\cdots x_{N-1}^{j_{N-1}}}$$

$$T^{\{\nu_0\cdots\nu_{N-1}\};\, d}_{\underbrace{0\cdots 0}_{2r}\underbrace{1\cdots 1}_{n_1}\cdots\underbrace{(N-1)\cdots(N-1)}_{n_{N-1}}} = \cancel{\mu^{2\epsilon}} \sum_{j_1\cdots j_{N-1}}^{\nu_0-1} \binom{\nu_0-1}{j_1,\ldots,j_{N-1},\,\nu_0-1-j_1-\ldots-j_{N-1}} \frac{\cancel{(-1)^{(\nu_0+\ldots+\nu_{N-1})+(n_1+\ldots+n_{N-1})+r+(j_1+\ldots+j_{N-1})}}}{\cancel{2^r}}$$

$$\cancel{\frac{e^{\gamma_E\epsilon}\,\Gamma(\nu_0+\ldots+\nu_{N-1}-\frac{d}{2}-r)}{\Gamma(\nu_0)\cdots\Gamma(\nu_{N-1})}} \int_0^1 dx_1\cdots dx_{N-1}\; \frac{x_1^{\nu_1+n_1-1+j_1}\cdots x_{N-1}^{\nu_{N-1}+n_{N-1}-1+j_{N-1}}}{\Delta^{-r+\frac{d}{2}+\nu_0+\ldots+\nu_{N-1}}}$$

Identify as $T_{0\cdots 0 1\cdots 1\cdots}$ with $n_1^{\mathrm{eff}} = \nu_1+n_1-1+j_1$

(unit weights for $d=4-2\epsilon$) $n_2^{\mathrm{eff}} = \nu_2+n_2-1+j_2$

$\vdots$

$$\nu_0+\ldots+\nu_{N-1}-\frac{\overset{D-2\epsilon}{\downarrow}}{\,}\!\!\frac{d}{2}-r = N-\frac{4-2\epsilon}{2}-r_{\mathrm{eff}}$$

$$\Rightarrow r^{\mathrm{eff}} = r-\frac{4-D}{2}+(N-\nu_0-\ldots-\nu_{N-1})$$

$$\int dx\; \frac{x_1^{n_1^{\mathrm{eff}}}\cdots x_{N-1}^{n_{N-1}^{\mathrm{eff}}}}{(\Delta)^{N-\frac{d}{2}-r_{\mathrm{eff}}}} = \left[\mu^{2\epsilon}\, \frac{(-1)^{N+n_1^{\mathrm{eff}}+\ldots+n_{N-1}^{\mathrm{eff}}+r_{\mathrm{eff}}}}{2^{r_{\mathrm{eff}}}}\, e^{\gamma_E\epsilon}\,\cancel{\Gamma(N-\tfrac{4-2\epsilon}{2}-r_{\mathrm{eff}})}\right]^{-1} (T_N)_{\underbrace{0\cdots 0 1\cdots 1\cdots}_{P}}$$

$$(-1)^\wedge\left[\cancel{N}+(\nu_1+\ldots+\nu_{N-1})+\cancel{(n_1+\ldots+n_{N-1})}-(N-1)+(j_1+\ldots+j_{N-1})+\cancel{r}-\frac{4-D}{2}+\cancel{N}-\cancel{(\nu_0+\ldots+\nu_{N-1})}\right]$$

$$2^{r_{\mathrm{eff}}} = 2^{r-\frac{4-D}{2}+N-(\nu_0+\ldots+\nu_{N-1})}$$

$$T^{\{\nu_0 \cdots \nu_{N-1}\};\, d=D-2\epsilon}_{\underbrace{0\cdots 0}_{2r}\,\underbrace{1\cdots 1}_{n_1}\cdots\underbrace{(N-1)\cdots(N-1)}_{n_{N-1}}} = \underbrace{\frac{(-1)^{N-1-(\nu_1+\ldots+\nu_{N-1})-\frac{4-D}{2}}}{2^{-N+(\nu_0+\ldots+\nu_{N-1})+(4-D)/2}}}_{(-1)^{-1+\nu_0}\frac{(-1)^{N-(\nu_0+\ldots+\nu_{N-1})-\frac{4-D}{2}}}{2^{-N+(\nu_0+\ldots+\nu_{N-1})+(4-D)/2}}} \frac{1}{\Gamma(\nu_0)\cdots\Gamma(\nu_{N-1})} \times \sum_{j_1\cdots j_{N-1}}^{\nu_0-1}{}' \begin{pmatrix} \nu_0-1 \\ j_1 \ldots \end{pmatrix} (T_N)_{0\ldots\ldots}$$

$$= (-1)^{\nu_0-1}(-2)^{N-(\nu_0+\ldots+\nu_{N-1})-(4-D)/2}$$

so,

$$T^{\{\nu_0,\ldots,\nu_{N-1}\};\, d=D-2\epsilon}_{\underbrace{0\cdots 0}_{2r}\,\underbrace{1\cdots 1}_{n_1}\cdots\underbrace{(N-1)\cdots(N-1)}_{n_{N-1}}} = \frac{(-1)^{\nu_0-1}(-2)^{N-(\nu_0+\cdots+\nu_{N-1})-(4-D)/2}}{\Gamma(\nu_0)\cdots\Gamma(\nu_{N-1})} \times \sum_{j_1\cdots j_{N-1}=0}^{\nu_0-1}{}' \begin{pmatrix} \nu_0-1 \\ j_1 \ldots \end{pmatrix} (T_N)_{\underbrace{0\cdots 0}_{2\left[r-\frac{4-D}{2}+N-(\nu_0+\ldots+\nu_{N-1})\right]}\,\underbrace{1\cdots 1}_{n_1+\nu_1-1+j_1}\cdots\underbrace{(N-1)\cdots(N-1)}_{-}}$$

$d = D - 4\epsilon$ $\quad$ $D \equiv$ integer part of number of spacetime dimensions

Special cases (valid if $\nu_0 \cdots \nu_{N-1} > 0$)

Coefficient functions in the right hand sides are all normal (unit weight in 4 dimensions)

$$A^{\{\nu_0\}d}_{\underbrace{0\cdots0}_{2r}}(m_0) = \frac{(-1)^{\nu_0-1}(-2)^{1-\nu_0-(4-D)/2}}{\Gamma(\nu_0)}\, A_{\underbrace{0\cdots0}_{2(r-\frac{4-D}{2}+1-\nu_0)}}(m_0)$$

$$B^{\{\nu_0,\nu_1\}d}_{\underbrace{0\cdots0}_{2r}\,\underbrace{1\cdots1}_{n_1}}(p^2, m_0, m_1) = \frac{(-1)^{\nu_0-1}(-2)^{2-\nu_0-\nu_1-(4-D)/2}}{\Gamma(\nu_0)\Gamma(\nu_1)} \sum_{j_1=0}^{\nu_0-1} \binom{\nu_0-1}{j_1} B_{\underbrace{0\cdots0}_{2(r-\frac{4-D}{2}+2-\nu_0-\nu_1)}\,\underbrace{1\cdots1}_{\nu_1+n_1-1+j_1}}(p^2, m_0, m_1)$$

$$C^{\{\nu_0,\nu_1,\nu_2\}d}_{\underbrace{0\cdots0}_{2r}\,\underbrace{1\cdots1}_{n_1}\,\underbrace{2\cdots2}_{n_2}}(p_1^2, q^2, p_2^2; m_0, m_1, m_2)$$

$$= \frac{(-1)^{\nu_0-1}(-2)^{3-\nu_0-\nu_1-\nu_2-(4-D)/2}}{\Gamma(\nu_0)\Gamma(\nu_1)\Gamma(\nu_2)} \sum_{j_1=0}^{\nu_0-1}\sum_{j_2=0}^{\nu_0-1-j_1} \binom{\nu_0-1}{j_1,\, j_2,\, \nu_0-1-j_1-j_2}$$

$$\times\; C_{\underbrace{0\cdots0}_{2(r-\frac{4-D}{2}+3-\nu_0-\nu_1-\nu_2)}\,\underbrace{1\cdots1}_{\nu_1+n_1-1+j_1}\,\underbrace{2\cdots2}_{\nu_2+n_2-1+j_2}}(p_1^2, q^2, p_2^2; m_0, m_1, m_2)$$

$$D^{\{\nu_0,\nu_1,\nu_2,\nu_3\}d}_{\underbrace{0\cdots0}_{2r}\,\underbrace{1\cdots1}_{n_1}\,\underbrace{2\cdots2}_{n_2}\,\underbrace{3\cdots3}_{n_3}}(s_1, s_2, s_3, s_4; s_{12}, s_{23}; m_0, m_1, m_2, m_3)$$

$$= \frac{(-1)^{\nu_0-1}(-2)^{4-\nu_0-\nu_1-\nu_2-\nu_3-(4-D)/2}}{\Gamma(\nu_0)\Gamma(\nu_1)\Gamma(\nu_2)\Gamma(\nu_3)} \sum_{j_1=0}^{\nu_0-1}\sum_{j_2=0}^{\nu_0-1-j_1}\sum_{j_3=0}^{\nu_0-1-j_1-j_2} \binom{\nu_0-1}{j_1,\, j_2,\, j_3,\, \nu_0-1-j_1-j_2-j_3}$$

$$\times\; D_{\underbrace{0\cdots0}_{2(r-\frac{4-D}{2}+4-\nu_0-\nu_1-\nu_2-\nu_3)}\,\underbrace{1\cdots1\;\;2\cdots2\;\;3\cdots3}_{\nu_i+n_i-1-j_i}}(s_1, s_2, s_3, s_4; s_{12}, s_{23}; m_0, m_1, m_2, m_3)$$