Short-circuiting identities

If one of the ν's is negative or zero, one of the propagators is missing.

$\longrightarrow$ short circuited.

Suppose k^{th} ν is non-positive ($\nu_k \le 0$):

$$T^{\{\dots\nu_k\dots\}}_{\underbrace{0\dots0}_{2r}\underbrace{1\dots1}_{n_1}\dots\underbrace{k\dots k}_{n_k}\dots} = \mu^{2\epsilon} e^{\gamma_E \epsilon} \frac{(-1)^{\nu_0+\dots\nu_k+\dots\nu_{N-1}+n_1+\dots+n_{N-1}+r}}{2^r} \frac{\Gamma(\nu_0+\dots\nu_k+\dots+\nu_{N-1}-\frac{d}{2}-r)}{\Gamma(\nu_0)\dots\Gamma(\nu_k)\dots\Gamma(\nu_{N-1})}$$

implies whole expression vanishes

$$\times\int_0^1 dx_1\dots dx_k\dots dx_N\; x_1^{\nu_1+n_1-1}\dots x_k^{\nu_k+n_k-1}\dots x_{N-1}^{\nu_{N-1}+n_{N-1}-1}\, x_N^{\nu_0-1}\,\delta(1-x_1-\dots-x_N)\left(\frac{1}{\Delta}\right)^{-r-\frac{d}{2}+\nu_0+\dots+\nu_{N-1}}$$

unless $\nu_k+n_k-1<0$:
integrand is singular as $x_k\to 0$
(sum over the other x's approach 1)

– The $\frac{1}{\Gamma(\nu_k)} \sim \#\nu_k$ implies whole expression vanishes.

If $\hat{\nu}_k + n_k = 0$, then the $\frac{1}{\Gamma(\nu_k)}$ zero balances the $\int dx_k \frac{1}{x_k^{1-(\nu_k+n_k)}}$ singularity.
Let $\hat{\nu}_k+n_k=\delta$, and investigate behavior as $\delta\to 0$:

$$\frac{1}{\Gamma(\nu_k)}\cdot\int_0^1 dx_k\, \frac{1}{x_k^{1-\delta}} \simeq \frac{1}{\frac{(-1)^{n_k}}{n_k!\,\delta}+\dots}\times\int_0^1 dx_k\left[\frac{\delta(x_k)}{\delta}+\dots\right]$$

$$= (-1)^{n_k}\, n_k! + O(\delta)$$

Therefore:

$$T^{\{\dots,-n_k,\dots\}}_{\underbrace{0\dots0}_{2r}\underbrace{1\dots1}_{n_1}\dots\underbrace{k\dots k}_{n_k}\dots} = \mu^{2\epsilon} e^{\gamma_E\epsilon}\frac{(-1)^{\nu_0+\dots-n_k+\dots\nu_{N-1}+n_1+\dots+n_k+\dots n_{N-1}+r}}{2^r}\frac{\Gamma(\nu_0+\dots-n_k+\dots+\nu_{N-1}-\frac{d}{2}-r)}{\Gamma(\nu_0)\dots\widehat{\Gamma(\nu_k)}\dots\Gamma(\nu_{N-1})}(-1)^{n_k}(n_k)!$$

$$\times\int_0^1 dx_1\dots\widehat{dx_k}\dots dx_N\; x_1^{\nu_1+n_1-1}\dots\widehat{x_k}\dots x_{N-1}^{\nu_{N-1}+n_{N-1}-1}\, x_N^{\nu_0-1}\,\delta(1-x_1-\dots\widehat{x_k}\dots x_N)\left(\frac{1}{\Delta(x_k=0)}\right)^{-r+\frac{d}{2}+\nu_0+\dots+\nu_{N-1}-n_k}$$

$\Rightarrow$ k^{th} propagator pinched. $\quad n_k^{new} = n_{k+1}$

$r_{new} = r + n_k \quad n_{k+1}^{new} = n_{k+2}$

$\vdots$

$$T^{\{\dots,-n_k,\dots\}}_{\underbrace{0\dots0}_{2r}\underbrace{1\dots1}_{n_1}\dots\underbrace{k\dots k}_{n_k}\dots} = \underbrace{(-2)^{n_k}\times(-1)^{n_k}(n_k)!}_{2^{n_k}(n_k)!}\; T^{\{\nu_0,\dots\widehat{\nu_k},\dots\nu_{N-1}\}}_{\underbrace{0\dots0}_{2(r+n_k)}\underbrace{1\dots1}_{n_1}\dots\underbrace{k\dots k}_{n_{k+1}}\underbrace{(k+1)\dots(k+1)}_{n_{k+2}}\dots\underbrace{(N-2)\dots(N-2)}_{n_{N-1}}}(\hat{D}_k)$$

Special cases: (all coefficients in $d = D - 2\epsilon$ dimensions)

Non-positive weights indicated by a hat like so: $\{\nu_0, \underset{\uparrow}{\hat{\nu}_1}, \nu_2\}$

$$A^{\{\nu_0\}}_{\underbrace{0\cdots0}_{2r}}(m_0) = 0 \quad \text{if } \nu_0 \le 0 .$$

$$B^{\{\hat{\nu}_0, \nu_1\}}_{\underbrace{0\cdots0}_{2r}\underbrace{1\cdots1}_{n_1}}(p^2; m_0, m_1) = \tilde{A}^{\{\nu_1\}}_{\underbrace{0\cdots0}_{2r}\underbrace{1\cdots1}_{n_1}}(m_1) \quad \text{if } \nu_0 = 0 \quad \left(\begin{matrix}\text{ill-defined}\\ \text{if } \nu_0 < 0\end{matrix}\right)$$

$$B^{\{\nu_0, \hat{\nu}_1\}}_{\underbrace{0\cdots0}_{2r}\underbrace{1\cdots1}_{n_1}}(p^2; m_0, m_1) = 2^{n_1} n_1!\, A^{\{\nu_0\}}_{\underbrace{0\cdots0}_{2(r+n_1)}}(m_0) \quad \text{if } n_1 = -\nu_1 \quad \left(\begin{matrix}0 \text{ if}\\ n_1 > -\nu_1\end{matrix}\right) \quad \left(\begin{matrix}\text{ill defined}\\ \text{if } n_1 < -\nu_1\end{matrix}\right)$$

$$C^{\{\hat{\nu}_0, \nu_1, \nu_2\}}_{\underbrace{0\cdots0}_{2r}\underbrace{1\cdots1}_{n_1}\underbrace{2\cdots2}_{n_2}}(p_1^2, q^2, p_2^2; m_0, m_1, m_2) = \tilde{B}^{\{\nu_1, \nu_2\}}_{\underbrace{0\cdots0}_{2r}\underbrace{1\cdots1}_{n_1}\underbrace{2\cdots2}_{n_2}}(q^2; m_1, m_2) \quad \text{if } \nu_0 = 0 \quad \left(\begin{matrix}\text{ill-defined}\\ \text{if } \nu_0 < 0\end{matrix}\right)$$

$$C^{\{\nu_0, \hat{\nu}_1, \nu_2\}}_{\underbrace{0\cdots0}_{2r}\underbrace{1\cdots1}_{n_1}\underbrace{2\cdots2}_{n_2}}(\text{— same —}) = 2^{n_1} n_1!\, B^{\{\nu_0, \nu_2\}}_{\underbrace{0\cdots0}_{2(r+n_1)}\underbrace{1\cdots1}_{n_2}}(p_2^2; m_0, m_2) \quad \text{if } n_2 = -\nu_2$$

$$C^{\{\nu_0, \nu_1, \hat{\nu}_2\}}_{0\cdots0\, 1\cdots1\, \underbrace{2\cdots2}_{n_2}}(\text{— same —}) = 2^{n_2} n_2!\, B^{\{\nu_0, \nu_1\}}_{\underbrace{0\cdots0}_{2(r+n_2)}\underbrace{1\cdots1}_{n_1}}(p_1^2; m_0, m_1) \quad \text{if } n_1 = -\nu_1$$

$$D^{\{\hat{\nu}_0, \nu_1, \nu_2, \nu_3\}}_{\underbrace{0\cdots0}_{2r}\underbrace{1\cdots1}_{n_1}\underbrace{2\cdots2}_{n_2}\underbrace{3\cdots3}_{n_3}}(s_1, s_2, s_3, s_4; s_{12}, s_{23}; m_0, m_1, m_2, m_3) = \tilde{C}^{\{\nu_1, \nu_2, \nu_3\}}_{0\cdots0\, 1\cdots1\, 2\cdots2\, 3\cdots3}(s_2, s_3, s_{23}; m_1, m_2, m_3)$$

$$\text{if } \nu_0 = 0 \quad \left(\begin{matrix}\text{ill-defined}\\ \text{if } \nu_0 < 0\end{matrix}\right)$$

$$D^{\{\nu_0, \hat{\nu}_1, \nu_2, \nu_3\}}_{0\cdots0\, \underbrace{1\cdots1}_{n_1}\, 2\cdots2\, 3\cdots3}(\text{— same —}) = 2^{n_1} n_1!\, C^{\{\nu_0, \nu_2, \nu_3\}}_{\underbrace{0\cdots0}_{2(r+n_1)}\underbrace{1\cdots1}_{n_2}\underbrace{2\cdots2}_{n_3}}(s_{12}, s_3, s_4; m_0, m_2, m_3)$$

$$\text{if } n_1 = -\nu_1$$

$$D^{\{\nu_0, \nu_1, \hat{\nu}_2, \nu_3\}}_{0\cdots0\, 1\cdots1\, \underbrace{2\cdots2}_{n_2}\, 3\cdots3}(\text{— same —}) = 2^{n_2} n_2!\, C^{\{\nu_0, \nu_1, \nu_3\}}_{\underbrace{0\cdots0}_{2(r+n_2)}\underbrace{1\cdots1}_{n_1}\underbrace{2\cdots2}_{n_3}}(s_1, s_{23}, s_4; m_0, m_1, m_3)$$

$$\text{if } n_2 = -\nu_2$$

$$D^{\{\nu_0, \nu_1, \nu_2, \hat{\nu}_3\}}_{0\cdots0\, 1\cdots1\, 2\cdots2\, \underbrace{3\cdots3}_{n_3}}(\text{— same —}) = 2^{n_2} n_3!\, C^{\{\nu_0, \nu_1, \nu_2\}}_{\underbrace{0\cdots0}_{2(r+n_3)}\underbrace{1\cdots1}_{n_1}\underbrace{2\cdots2}_{n_2}}(s_1, s_2, s_{12}; m_0, m_1, m_2)$$

$$\text{if } n_3 = -\nu_3$$