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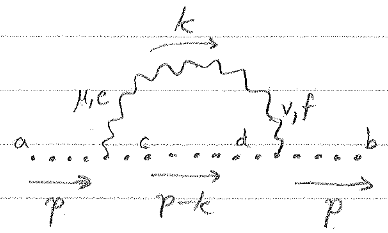
Ghost Self Energy



Count contractions

possible vertices for ghost to contract to.

$$\langle 0|\eta \;\; \partial\bar{\eta}\eta A \;\; \partial\bar{\eta}\eta A \;\; \bar{\eta}|0\rangle = 2$$

Taylor factor $= \frac{1}{2!}$

$$-i\Sigma(p^2) = \int \frac{d^dk}{(2\pi)^d}\left(-g\mu^\epsilon f^{fdb}(p-k)_\nu\right)\frac{-i\delta^{fe}}{k^2}\left(g^{\nu\mu} - (1-\xi)\frac{k^\nu k^\mu}{k^2}\right)\frac{i\delta^{dc}}{(q-k)^2}\left(-g\mu^\epsilon f^{eac}\, p_\mu\right)$$

Factor out i's and $g\mu^\epsilon$'s: $+g^2\mu^{2\epsilon}$

Color tensor: $f^{fdb}\delta^{fe}\delta^{de}f^{eac} = f^{dcb}f^{d\overset{\frown}{ac}} = -C_A\delta^{ab}$

$$= -g^2\mu^{2\epsilon}C_A\delta^{ab}\int\frac{d^dk}{(2\pi)^d}\frac{(p-k)_\nu\, p_\mu}{k^2(p-k)^2}\left(g^{\mu\nu} - (1-\xi)\frac{k^\mu k^\nu}{k^2}\right)$$

Open up the integral:

$$= -g^2\mu^{2\epsilon}C_A\delta^{ab}\Bigg(g^{\mu\nu}\left[p_\mu p_\nu\int\frac{d^dk}{(2\pi)^d}\frac{1}{k^2(p-k)^2} - p_\mu\int\frac{d^dk}{(2\pi)^d}\frac{k_\nu}{k^2(p-k)^2}\right]$$
$$-(1-\xi)\left[p_\mu p_\nu\int\frac{d^dk}{(2\pi)^d}\frac{k^\mu k^\nu}{k^2(p-k)^2k^2} - p_\mu\int\frac{d^dk}{(2\pi)^d}\frac{k_\nu k^\mu k^\nu}{k^2(p-k)^2k^2}\right]\Bigg)$$

$$= -g^2\mu^{2\epsilon}C_A\delta^{ab}\left[\left(\frac{ip^2}{(4\pi)^2 2} - (1-\xi)\frac{-ip^2}{(4\pi)^2 4}\right)\left(\frac{1}{\epsilon} - \gamma_E + \ln 4\pi - \ln\left(\frac{-p^2}{\mu^2}\right)\right)\right.$$
$$\left. + \frac{-ip^2}{(4\pi)^2 2}\times 2\right]$$

$$= i\delta^{ab}p^2\frac{g^2}{(4\pi)^2}\left[-\frac{3-\xi}{4}C_A\left(\frac{1}{\epsilon} - \gamma_E + \ln 4\pi - \ln\left(\frac{-p^2}{\mu^2}\right)\right) + C_A\right]$$

Add counterterm: $a \overset{p}{\cdots\otimes\cdots} b = i\delta^{ab}p^2\delta_\eta$

$\overline{MS}$ renormalization: $\delta_\eta = \frac{g^2}{(4\pi)^2}\left(\frac{3-\xi}{4}C_A\right)\left(\frac{1}{\epsilon} - \gamma_E + \ln 4\pi\right)$