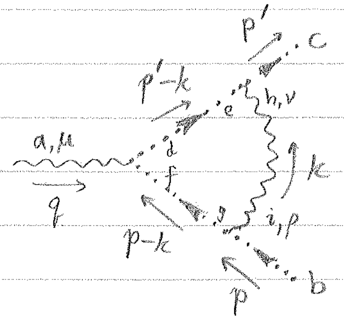
Ghost-Gluon Vertex — GRAPH 1



Contractions:

ext. A_μ field contr. ext. η & $\bar\eta$ contr.

$$\langle \eta\bar\eta \quad \partial\bar\eta\eta A \quad \partial\bar\eta\eta A \quad \partial\bar\eta\eta A \quad A\rangle = 3\times 2 = 6$$

$$\text{Taylor expansion} = 1/3! = 1/6$$

Kinematics: $q = p' - p$

$$g\mu^\epsilon \Gamma^{abc}_\mu = \int \frac{d^dk}{(2\pi)^d}\left(-g\mu^\epsilon f^{afd}(p-k)_\mu\right)\frac{i\delta^{de}}{(p'-k)^2}\left(g\mu^\epsilon f^{hec}(p'-k)_\nu\right)\frac{-i\delta^{hi}}{k^2}\left(-g^{\nu\rho}-(1-\xi)\frac{k^\nu k^\rho}{k^2}\right)$$
$$\times\left(g\mu^\epsilon f^{ibg}p_\rho'\right)\frac{i\delta^{fg}}{(p-k)^2}$$

Factor out i's and $g\mu^\epsilon$'s: $ig^3\mu^{3\epsilon} = g\mu^\epsilon(ig^2\mu^{2\epsilon})$

Simplify color structure: $f^{afd}\delta^{de}f^{hec}\delta^{hi}f^{ibg}\delta^{fg} = f^{agd}f^{hdc}f^{hbg} = \frac{C_A}{2}f^{abc}$

$$= g\mu^\epsilon\left[-ig^2\mu^{2\epsilon}f^{abc}\frac{C_A}{2}\int\frac{d^dk}{(2\pi)^d}\frac{(p-k)_\mu(p'-k)_\nu p_\rho'}{(p'-k)^2(p-k)^2k^2}\left(g^{\nu\rho}-(1-\xi)\frac{k^\nu k^\rho}{k^2}\right)\right]$$

only $k_\mu k_\nu$ piece in here contains divergence.

Mathematica

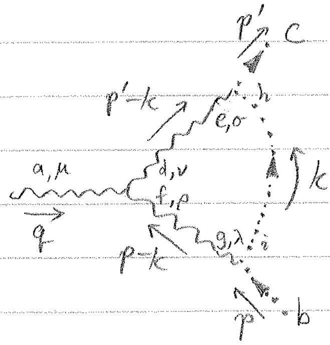
$$\Gamma^{abc}_\mu = -ig^2\mu^{2\epsilon}f^{abc}\frac{C_A}{2}p_\rho\int\frac{d^dk}{(2\pi)^d}\frac{k_\mu k_\nu}{(p'-k)^2(p-k)^2k^2}\left(g^{\nu\rho}-(1-\xi)\frac{k^\nu k^\rho}{k^2}\right)$$

Integrate (with my Mathematica code), to obtain the divergent terms:

$$= -ig^2 f^{abc}\frac{C_A}{2}\left(\frac{ip^\mu}{(4\pi)^2 4}-(1-\xi)\frac{ip^\mu}{(4\pi)^2 4}\right)\left(\frac{1}{\epsilon}-\gamma_E+\ln 4\pi\right)$$

$$= f^{abc}p^\mu\left[\frac{g^2}{(4\pi)^2}\frac{\xi C_A}{8}\left(\frac{1}{\epsilon}-\gamma_E+\ln 4\pi\right)+\text{finite}\right]$$

GRAPH 2



Contractions

ext. ghost contractions

$$\langle \eta\bar{\eta} \quad \partial\bar{\eta}\eta A \quad \partial\bar{\eta}\eta A \quad AAA \quad A\rangle = 2$$

Different contractions here already taken into account by Feyn. rule.

Taylor expansion $= \frac{1}{3!}$ & $(\times 3)$ due to cross term.

$$g\mu^\epsilon \Gamma^{abc}_\mu = \int \frac{d^dk}{(2\pi)^d}\left(-g\mu^\epsilon f^{ehc}k_\sigma\right)\frac{i\delta^{hi}}{k^2}\left(-g\mu^\epsilon f^{gbi}p_\lambda\right)\frac{-i\delta^{fg}}{(p-k)^2}\left(g^{\rho\lambda} - (1-\xi)\frac{(p-k)^\rho(p-k)^\lambda}{(p-k)^2}\right)$$

$$\times\left(-g\mu^\epsilon\right)f^{adf}\left(g_{\mu\nu}(q+p'-k)_\rho + g_{\nu\rho}(-p'+k-p+k)_\mu + g_{\rho\mu}(p-k-q)_\nu\right)$$

$$\times\frac{-i\delta^{de}}{(p'-k)^2}\left(g^{\nu\sigma} - (1-\xi)\frac{(p'-k)^\nu(p'-k)^\sigma}{(p'-k)^2}\right)$$

Factor out i's and $g\mu^\epsilon$'s : $ig^3\mu^{3\epsilon} = g\mu^\epsilon(ig^2\mu^{2\epsilon})$

Simplify color structure: $f^{ehc}\delta^{hi}f^{gbi}\delta^{fg}f^{adf}\delta^{de} = f^{dhc}f^{fbh}f^{adf} = \frac{C_A}{2}f^{abc}$

$$g\mu^\epsilon\Gamma^{abc}_\mu = g\mu^\epsilon\left\{ig^2\mu^{2\epsilon}f^{abc}\frac{C_A}{2}\int\frac{d^dk}{(2\pi)^d}\frac{k_\sigma p_\lambda\left[g_{\mu\nu}(q+p'-k)_\rho + g_{\nu\rho}(-p'+2k-p)_\mu + g_{\rho\mu}(p-k-q)_\nu\right]}{(p'-k)^2(p-k)^2k^2}\right.$$

$$\left.\times\left(g^{\rho\lambda} - (1-\xi)\frac{(p-k)^\rho(p-k)^\lambda}{(p-k)^2}\right)\left(g^{\nu\sigma} - (1-\xi)\frac{(p'-k)^\nu(p'-k)^\sigma}{(p'-k)^2}\right)\right\}$$

Simplify 3-gluon vertex factor, $[\ldots]$

$$[\ldots] = g_{\mu\nu}(-p+2p'-k)_\rho + g_{\nu\rho}(-p-p'+2k)_\mu + g_{\rho\mu}(2p-p'-k)_\nu$$

$$= \left(-g_{\mu\nu}g_{\rho\delta} - g_{\nu\rho}g_{\mu\delta} + 2g_{\rho\mu}g_{\nu\delta}\right)p^\delta + \left(2g_{\mu\nu}g_{\rho\delta} - g_{\nu\rho}g_{\mu\delta} - g_{\rho\mu}g_{\nu\delta}\right)p'^\delta$$

$$+\left(-g_{\mu\nu}g_{\rho\delta} + 2g_{\nu\rho}g_{\mu\delta} - g_{\rho\mu}g_{\nu\delta}\right)k^\delta$$

$$= G_{\rho\mu\nu\delta}\,p^\delta + G_{\mu\nu\rho\delta}\,p'^\delta + G_{\nu\rho\mu\delta}\,k^\delta$$

defined in vac. polarization computations.

So, Γ^{abc}_μ is:

$$\Gamma^{abc}_\mu = i g^2 \mu^{2\epsilon} f^{abc}\frac{C_A}{2}\, p_\lambda \int\frac{d^dk}{(2\pi)^d}\left(\frac{k_\sigma\left[G_{\rho\mu\nu\delta}\,p^\delta + G_{\mu\nu\rho\delta}\,p'^\delta + G_{\nu\rho\mu\delta}\,k^\delta\right]}{(p'-k)^2(p-k)^2k^2}\right.$$
$$\left.\times\left(g^{\rho\lambda}-(1-\xi)\frac{(p-k)^\rho(p-k)^\lambda}{(p-k)^2}\right)\left(g^{\nu\sigma}-(1-\xi)\frac{(p'-k)^\nu(p'-k)^\sigma}{(p'-k)^2}\right)\right)$$

In the first factor, the only divergent term is the one containing $G_{\nu\rho\mu\delta}k^\delta$ (count powers of k: 2 in num., 6 in denom $\Rightarrow$ log. div).

In the gauge boson propagators, divergences in the ξ-dep terms come from $k^\rho k^\lambda$ and $k^\nu k^\sigma$ terms only.

So, the divergent part of Γ^{abc}_μ is:

$$\Gamma^{abc}_\mu\Big|_{DIV} = i g^2 \mu^{2\epsilon} f^{abc}\frac{C_A}{2}\, p_\lambda \int\frac{d^dk}{(2\pi)^d}\left[\frac{G_{\nu\rho\mu\delta}\,k^\delta k_\sigma}{(p'-k)^2(p-k)^2k^2}\left(g^{\rho\lambda}-(1-\xi)\frac{k^\rho k^\lambda}{(p-k)^2}\right)\left(g^{\nu\sigma}-(1-\xi)\frac{k^\nu k^\sigma}{(p'-k)^2}\right)\right]$$

In Mathematica

Integrals performed with the help of Mathematica — 4 terms:

$$\left(g^{\rho\lambda}-(1-\xi)A^{\rho\lambda}\right)\times\left(g^{\nu\sigma}-(1-\xi)B^{\nu\sigma}\right)$$ (Two vanish - due to Lorentz struc.)

$$= i g^2 f^{abc}\frac{C_A}{2}\left[\frac{-3ip^\mu}{(4\pi)^2 4} + (1-\xi)\frac{3ip^\mu}{(4\pi)^2 4} + 0 + 0\right]\left(\frac{1}{\epsilon}-\gamma_E+\ln 4\pi\right)$$

$$\Gamma^{abc}_\mu = f^{abc}p^\mu\left[\frac{g^2}{(4\pi)^2}\,\frac{3\xi}{8}\,C_A\left(\frac{1}{\epsilon}-\gamma_E+\ln 4\pi\right)+\text{finite}\right]$$

Counterterm:  $= \delta_{\eta\eta A}\,\mu^\epsilon f^{abc}p^\mu \quad\Rightarrow\quad \Gamma^{abc}_{CT\,\mu} = f^{abc}p^\mu\,\frac{\delta_{\eta\eta A}}{g}$

So, together: $$\Gamma^{abc}_\mu = f^{abc}p^\mu\left[\frac{g^2}{(4\pi)^2}\underbrace{\left(\frac{\xi}{8}+\frac{3\xi}{8}\right)}_{\frac{1}{2}\xi}C_A\left(\frac{1}{\epsilon}-\gamma_E+\ln 4\pi\right)+\frac{\delta_{\eta\eta A}}{g}+(\text{finite})\right.$$

So, $\overline{MS}$: $$\frac{\delta_{\eta\eta A}}{g} = \frac{g^2}{(4\pi)^2}\left(\frac{-\xi}{2}C_A\right)\left(\frac{1}{\epsilon}-\gamma_E+\ln 4\pi\right)$$