

## Setting Up

To  $\mathcal{O}(\alpha_g)$

$$i\mathcal{M}_{ee \rightarrow q\bar{q}} = \text{[tree-level diagram]} + \text{[gluon emission from } e^+ \text{]} + \text{[gluon emission from } e^- \text{]} + \text{[gluon emission from } q \text{]} + \text{[gluon emission from } \bar{q} \text{]} + \mathcal{O}(\alpha_g^2)$$

To cancel infrared divergences, we must include the following bremsstrahlung diagrams:

$$i\mathcal{M}_{ee \rightarrow q\bar{q}g} = \text{[gluon emission from } e^+ \text{]} + \text{[gluon emission from } e^- \text{]} + \mathcal{O}(g^3)$$

The full  $e^+e^- \rightarrow q\bar{q}$  or  $q\bar{q}g$  cross section,

$$d\sigma_{\text{Full}} = \frac{1}{(\text{Flux})} [ |\mathcal{M}_{ee \rightarrow q\bar{q}}|^2 d(\text{LIPS}_2) + |\mathcal{M}_{ee \rightarrow q\bar{q}g}|^2 d(\text{LIPS}_3) ] \quad (1)$$

involves the matrix element squares, given by

$$\begin{aligned} |\mathcal{M}_{ee \rightarrow q\bar{q}}|^2 &= \left| \text{[tree-level]} + \text{[gluon emission from } e^+ \text{]} + \text{[gluon emission from } e^- \text{]} + \text{[gluon emission from } q \text{]} + \text{[gluon emission from } \bar{q} \text{]} \right|^2 \\ &= \left| \text{[tree-level]} \right|^2 + 2\text{Re} \left[ \text{[tree-level]} \left( \text{[gluon emission from } e^+ \text{]} + \text{[gluon emission from } e^- \text{]} + \text{[gluon emission from } q \text{]} + \text{[gluon emission from } \bar{q} \text{]} \right) \right] + \mathcal{O}(\alpha^2) \end{aligned}$$

and

$$|\mathcal{M}_{ee \rightarrow q\bar{q}g}|^2 = \left| \text{[gluon emission from } e^+ \text{]} + \text{[gluon emission from } e^- \text{]} \right|^2$$