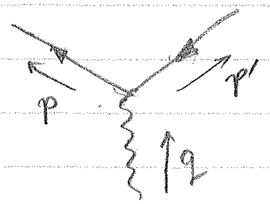
$$\cdots = \frac{1}{q^4} e^2 L_{\mu\nu} e^2 H^{\mu\nu}$$



$$q = (p+p') \Rightarrow q^2 = 2p\cdot p'$$

$$i\mathcal{M} = \bar{u}_i^{(s)}(p)\left(i Q_f e\mu^\epsilon \gamma^\mu \delta_{ij}\right) v_j^{(s')}(p')$$

$$(e\mu^\epsilon)^2 H^{\mu\nu} = |\mathcal{M}|^2 = \sum_{\substack{\text{flavor,}\\ f}} \sum_{\substack{\text{spins,}\\ s,s'}} \sum_{\substack{\text{colors}\\ c}} (e\mu^\epsilon)^2 Q_f^2\, \bar{u}_i^{(s)}(p)\gamma^\mu v_i^{(s')}(p')\bar{v}_j^{(s')}(p')\gamma^\nu u_j^{(s)}(p$$

$$= (e\mu^\epsilon)^2 \sum_{\text{flavor, } f} Q_f^2 \sum_{\text{colors}} \mathrm{Tr}\Big[\sum_s u_j^{(s)}(p)\bar{u}_i^{(s)}(p)\gamma^\mu v_j^{(s')}(p')\bar{v}_i^{(s')}(p')\gamma^\nu$$

$$= (e\mu^\epsilon)^2 \sum_f Q_f^2\ \mathrm{Tr}\left[\delta_{ij}\, \not{p}\, \gamma^\mu \delta_{ji}\, \not{p}'\gamma^\nu\right]$$

$$= (e\mu^\epsilon)^2 N_c \sum_f Q_f^2\ \mathrm{Tr}\left[\not{p}\,\gamma^\mu \not{p}'\gamma^\nu\right]$$

$$= (e\mu^\epsilon)^2 N_c \sum_f Q_f^2\ 4\left(p^\mu p'^\nu + p^\nu p'^\mu - (p\cdot p')g^{\mu\nu}\right)$$

Note: $q_\mu H^{\mu\nu} \sim (q\cdot p)p'^\nu + p^\nu(q\cdot p') - (p\cdot p')q^\nu \qquad q = p+p'$

$$(p\cdot p')p'^\nu + p^\nu(p\cdot p') - (p\cdot p')q^\nu$$

$$\cancel{(p\cdot p')(p'^\nu + p^\nu)} - \cancel{(p\cdot p')q^\nu} = 0.$$

Thus, using $\langle L_{\mu\nu}\rangle = \frac{1}{d-1}\left(-q^2 g_{\mu\nu} + q_\mu q_\nu\right)$, we have

$$|\mathcal{M}|^2 = \frac{1}{q^4}(e\mu^\epsilon)^2 L_{\mu\nu}(e\mu^\epsilon)^2 H^{\mu\nu} \qquad \text{Dropping } \sim q_\mu$$

$$= \frac{(e\mu^\epsilon)^4}{q^4}\frac{4}{3-2\epsilon} N_c \sum_f Q_f^2 \left(-q^2 g_{\mu\nu}\right)\left(p^\mu p'^\nu + p^\nu p'^\mu - (p\cdot p')g^{\mu\nu}\right)$$

$$= \frac{(e\mu^\epsilon)^4}{q^2}\frac{4}{3-2\epsilon} N_c \sum_f Q_f^2 \left(-p\cdot p' - p\cdot p' + d(p\cdot p')\right)$$

$$= \frac{(e\mu^\epsilon)^4}{q^2}\frac{4}{3-2\epsilon} N_c \sum_f Q_f^2 \underbrace{(d-2)}_{2(1-\epsilon)}\underbrace{p\cdot p'}_{q^2/2}$$

Now $\sigma = \frac{1}{\text{Flux}} |\mathcal{M}|^2 \, d(\text{LIPS})_2$

$$\text{Flux} = 4 (p_{e^+})^\mu (p_{e^-})_\mu = 2q^2$$

$$\int d(\text{LIPS})_2 = \frac{1}{4\pi} \frac{1}{2} \left(\frac{4\pi}{q^2}\right)^\epsilon \frac{1}{\cancel{\Gamma(1-\epsilon)}} \frac{\cancel{\Gamma(1-\epsilon)}\Gamma(1-\epsilon)}{\Gamma(2-2\epsilon)}$$

Hence, the born-level cross section is:

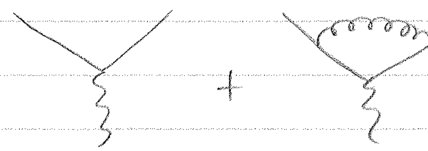
$$\sigma_{\text{born}} = \frac{1}{\cancel{2}q^2} \frac{(e\mu^\epsilon)^4}{\cancel{q^2}} \frac{4 \cdot \cancel{2}(1-\epsilon)}{3-2\epsilon} \sum_f Q_f^2 \frac{\cancel{q^2}}{\cancel{2}} \frac{1}{4\pi} \frac{1}{\cancel{2}} \left(\frac{4\pi}{q^2}\right)^\epsilon \frac{\Gamma(1-\epsilon)}{\Gamma(2-2\epsilon)}$$

Put $\mu^{2\epsilon}$ inside $\left(\frac{4\pi}{q^2}\right)^\epsilon$. Bring $1/4\pi$ in front.

$$= \frac{e^4 \mu^{2\epsilon}}{4\pi q^2} N_c \sum_f Q_f^2 \left(\frac{4\pi\mu^2}{q^2}\right)^\epsilon \frac{1-\epsilon}{3-2\epsilon} \frac{\Gamma(1-\epsilon)}{\Gamma(2-2\epsilon)}$$

$e^2/4\pi = \alpha_{em} \Rightarrow (4\pi)^2 \alpha_{em}^2 = e^4$

$$= \frac{4\pi \alpha_{em}^2 \mu^{2\epsilon}}{q^2} N_c \sum_f Q_f^2 \frac{1-\epsilon}{3-2\epsilon} \frac{\Gamma(1-\epsilon)}{\Gamma(2-2\epsilon)} \left(\frac{4\pi\mu^2}{q^2}\right)^\epsilon$$



$$= \bar{u}^{(s)}_i(p)\; i Q_f e \left[1 + \delta F_1(q^2)\right] \gamma^\mu v^{(s')}_i(p')$$

$$d\sigma = \frac{1}{\text{Flux}} |\mathcal{M}|^2 \, d(\text{LIPS})$$

$$|\mathcal{M}|^2 = \frac{1}{q^4} e^2 \langle L_{\mu\nu} \rangle e^2 H^{\mu\nu}$$

$$\langle L_{\mu\nu} \rangle = \frac{1}{d-1}\left(-q^2 g_{\mu\nu} + q_\mu q_\nu\right)$$

$$H^{\mu\nu} = \sum_{s,s'} \sum_{\text{colors}} Q_f^2 \, \bar{u}^{(s)}_i(p) \left[1 + \delta F_1(q^2)\right] \gamma^\mu v^{(s')}_i(p') \bar{v}^{(s')}_j(p') \left[1 + \delta F_1(q^2)\right]^* \gamma^\nu u^{(s)}_j$$

$$= Q_f^2 N_c^2 \left|1 + \delta F_1(q^2)\right|^2 \, \mathrm{Tr}\left[\not{p}\gamma^\mu \not{p}' \gamma^\nu\right] \delta_{ij}\delta_{ij}$$

$$= Q_f^2 N_c \left(1 + 2\mathrm{Re}[\delta F_1(q^2)]\right) 4 \left(p^\mu p'^\nu + p^\nu p'^\mu - g^{\mu\nu} p\cdot p'\right)$$

$$\sigma = \sigma_0 \left(1 + 2\,\mathrm{Re}[\delta F_1(q^2)]\right)$$

Then,

$$\langle L_{\mu\nu} \rangle \cdot H^{\mu\nu} = \frac{4 Q_f^2 N_c}{d-1}\left(1 + 2\mathrm{Re}[\delta F_1(q^2)]\right)\Big(-q^2 p\cdot p' - q^2 p\cdot p' + q^2 d\, p\cdot p'$$
$$+ (p\cdot q)(p'\cdot q) + (p'\cdot q)(p\cdot q) - q^2 p\cdot p'\Big)$$

$$= \frac{4 Q_f^2 N_c}{d-1}\left(1 - 2\mathrm{Re}[\delta F_1(q^2)]\right)\left((d-3) q^2 p\cdot p' + 2 (p\cdot q)(p'\cdot q)\right)$$

$\text{Flux} = 4 (p_{e^+})^\mu (p_{e^-})_\mu = 2q^2$ for massless incoming leptons

And $d(\text{LIPS})_2 = \frac{1}{4\pi}\,\frac{1}{2}\left(\frac{\pi}{s/4}\right)^\epsilon \frac{1}{\Gamma(1-\epsilon)}$