<u>Matrix-valued gauge fields</u>

Define: $A_\mu(x)_{ij} = A_\mu^a(x) T^a_{ij}$ (matrix-valued gauge field).

Then we would like to find a transformation law which recovers

$$A_\mu^a \longrightarrow A_\mu^a - \frac{1}{g}\partial_\mu \alpha^a - f^{abc}\alpha^b A_\mu^c + \mathcal{O}(\alpha^2)$$

The correct finite transformation law is:

$$\boxed{A_\mu \longrightarrow U(x) A_\mu U^\dagger(x) - \frac{i}{g} U(x)\partial_\mu U^\dagger(x).} \quad \text{with } U(x) = e^{i\alpha^a(x) T^a}$$

<u>Check:</u> (Exercise)

$$= e^{i\alpha^a T^a} A_\mu e^{-i\alpha^a T^a} - \frac{i}{g} e^{i\alpha^a T^a}\partial_\mu e^{-i\alpha^a T^a}$$

$$\simeq (1 + i\alpha^b T^b + \ldots) A_\mu^c T^c (1 - i\alpha^a T^a + \ldots)$$
$$- \frac{i}{g}(1 + i\alpha^a T^a + \ldots)\partial_\mu(1 - i\alpha^b T^b + \ldots)$$

$$= A_\mu^c T^c + i\alpha^b A_\mu^c \underbrace{[T^b, T^c]}_{if^{abc}T^a} - \frac{i}{g}(-i\,\partial_\mu \alpha^b T^b) + \mathcal{O}(\alpha^2)$$

rename: $c \to a$ rename $b \to a$

$$A_\mu^a T^a \to A_\mu^a T^a - \frac{1}{g}\partial_\mu \alpha^a T^a - f^{abc}\alpha^b A_\mu^c T^a \quad ✓$$

since T^a's are linearly independent, we can drop them, and we recover the original transformation law.

Note: $A_\mu^a \to \left(e^{i\alpha^d T^d_{Adj.}}\right)^{ab} A_\mu^b$ gives homogeneous part (w/o derivatives).

$$(T^d)_{ab} \equiv -if^{dab}$$

Field strength:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu] \quad \text{and} \quad F \to UFU^\dagger$$