Thus,

$$\sigma_{q\bar{q}g} = \sigma_{\text{born}}\frac{\alpha_s C_F}{2\pi}\left(\frac{2}{\epsilon^2} + \frac{1}{\epsilon}\left[3 - 2\gamma_E + 2\ln\left(\frac{4\pi\mu^2}{q^2}\right)\right]\right.$$
$$\left. + \left[\frac{19}{2} - 3\gamma_E - \frac{7\pi^2}{6} + \gamma_E^2 + (3-2\gamma_E)\ln\left(\frac{4\pi\mu^2}{q^2}\right) + \ln^2\left(\frac{4\pi\mu^2}{q^2}\right)\right] + O(\epsilon)\right)$$

Double pole $\frac{2}{\epsilon^2}$ from interference : soft gluon emission

Single pole $\frac{1}{\epsilon}\left[3 - 2\gamma_E + 2\ln\left(\frac{4\pi\mu^2}{q^2}\right)\right]$: collinear emission.

Adding virtual corrections to real emission:

$$\sigma_{q\bar{q}} = \sigma_{\text{born}}\left(1 + 2\,\text{Re}\left[\delta F_1(q^2)\right]\right)$$
$$= \sigma_{\text{born}}\left(1 + \cancel{2}\frac{C_F\alpha_s}{\cancel{4}_{2}\pi}\text{Re}\left(\frac{-2}{\epsilon^2} + \ldots\right)\right)$$
$$= \sigma_{\text{born}}\left(1 + \frac{\alpha_s C_F}{2\pi}\left(\frac{-2}{\epsilon^2} + \frac{1}{\epsilon}\left[-3 + 2\gamma_E - 2\ln\left(\frac{4\pi\mu^2}{q^2}\right)\right]\right.\right.$$
$$\left.\left. + \left[-8 + 3\gamma_E + \frac{7\pi^2}{6} - \gamma_E^2 - (3-2\gamma_E)\ln\left(\frac{4\pi\mu^2}{q^2}\right) - \ln\left(\frac{4\pi\mu^2}{q^2}\right)\right] + O(\epsilon)\right)\right)$$

The total (inclusive) cross section to $O(\alpha_s)$ is finite:

$$\sigma_{e^+e^-\to\text{had}} = \sigma_{q\bar{q}} + \sigma_{q\bar{q}g}$$
$$= \sigma_{\text{born}}\left[1 + \frac{\alpha_s C_F}{2\pi}\left(\frac{3}{2}\right) + O(\epsilon) + O(\alpha_s^2)\right]$$
$$= \sigma_{\text{born}}\left[1 + \frac{3C_F}{4}\frac{\alpha_s}{\pi} + O(\alpha_s^2)\right]$$

In QCD, $C_F = 4/3$. Thus $\sigma_{e^+e^-\to\text{had}} = \sigma_{\text{born}}\left[1 + \frac{\alpha_s}{\pi} + O(\alpha_s^2)\right]$

Taking $\alpha_s(1\text{ GeV}) \sim 0.3$
$\alpha_s(100\text{ GeV}) \sim 0.1$

$$= \sigma_{\text{born}}\left[1 + \begin{pmatrix}0.09\\0.03\end{pmatrix} + O(\alpha_s^2)\right]$$ up to 9% corr.